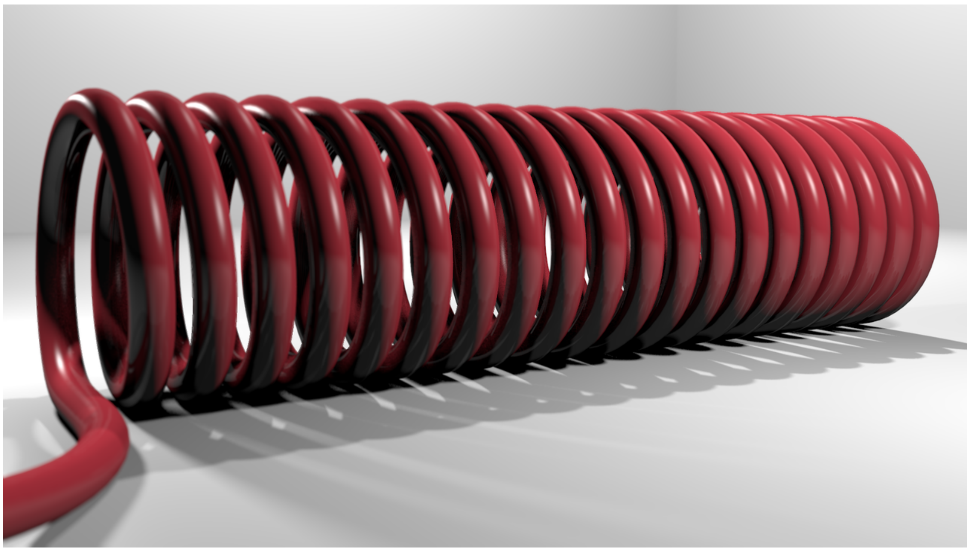
Week 9 - Solenoids, Ampère and Maxwell's 4th equation



Ampere was the Newton of Electricity.

James C. Maxwell

Gauss' law is to electricity what *Ampère's Law* is to magnetism. Ampère's law states that currents tend to cause circulating magnetic fields,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

We can turn it into it's integral equivalent by using *Stoke's theorem* on the left side of the following equation:

$$\int_{\mathcal{S}} \nabla \times \mathbf{B} \cdot d\mathbf{A} = \mu_0 \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{A}$$

Which gives

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{A} = \mu_0 I_{enclosed}$$

where the integration is taken around a *closed* loop defining the boundary of the surface $\mathcal{S}$.

Like Gauss' law, Ampère's law is a fundamental statement about static electromagnetic fields that always holds, but it isn't always useful. Only in cases with great symmetry it can be used to deduce fields that would otherwise be very complicated to find directly from Biot-Savart.

Exercise 9.1: Selected exercises from the book

Do exercise 23.19 and 23.20 from Generell Fysikk.

Answer: 23.19:

$$B(r < R) = \frac{\mu_0 I}{2\pi R^2} r$$

23.20: a) $I_{max} = 5.34 \times 10^{-3}$ A b) $B_{max} = 2.67 \times 10^{-9}$ T c) 8.5 V

Solution: 23.19:

The E-field from the condensator is $E = q/(\pi R^2 \epsilon_0)$, giving a flux through a surface S equal to

$$\Phi_S((E) = \frac{q\pi r^2}{\pi R^2} = \frac{qr^2}{R^2}$$

Looking at the situation for $r < R$ gives us

$$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial \Phi_S(\mathbf{E})}{\partial t} \quad (1)$$

$$B2\pi r = \mu_0 \varepsilon_0 \frac{\partial \Phi_S(\mathbf{E})}{\partial t} \quad (2)$$

$$B2\pi r = \mu_0 \varepsilon_0 \frac{r^2 \partial(\mathbf{q})}{R^2 \partial t} \quad (3)$$

$$B = \frac{\mu_0 I r}{2\pi R^2} \quad (4)$$

For more information see page 339 in Generell Fysikk.

Exercise 9.2: Field from an infinite cylinder

A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire if

a) The current is uniformly distributed over the outside surface of the wire.

Solution:

$$\int \mathbf{B}\mathrm{d}\mathbf{l} = \mu_0 I \quad (5)$$

$$B2\pi r = \mu_0 I \quad (6)$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (7)$$

Inside the wire the mangetic field is zero since no current is enclosed.

Answer: Outside:

$$B = \frac{\mu_0 I}{2\pi r}$$

Inside:

$$B = 0$$

b) The current is distributed such that $J = kr$.

Answer: Outside:

$$B = \frac{\mu_0 I}{2\pi r}$$

Inside:

$$B = \frac{\mu_0 k r^2}{3}$$

Exercise 9.3: The solenoid

In many applications we require constant electric and magnetic fields. The easiest way to generate constant electic fields is to use a parallel plate capacitor. To create a constanct magnetic field we do on the other hand use a *solenoid*.

a) Consider a cylinder of radius a wound up by a wire carrying a current I. The wire is wound with n_1 turns per unit length. Find the magnetic field inside the solenoid.

Solution: Use Ampère's law and choose a rectangular integration curve with the long side pointing along the axis of the solenoid. This gives nl windings with a total current of $I_{tot} = nlI$. We only need to look at the contributions inside the solenoid, resulting in the equation

$$\oint \mathbf{B} \cdot d\mathbf{l} = Bl = \mu_0 n I l$$

See page 337 in Generell Fysikk for more information

Answer:

$$B = \mu_0 n_1 I$$

b) Suppose we introduce yet another solenoid with $b > a$, coaxial with the first one and with n_2 turns per unit length, but with current running in the opposite direction. Find the field everywhere.

Exercise 9.4: Selected exercises from the book and exams

Do exercise 26.1 and 26.3 from Generell Fysikk.

Do exercise 4 from the 2009 exam. Exams from previous years are posted here:

http://www.uio.no/studier/emner/matnat/fys/FYS1120/h11/undervisningsmateriale/eksamensoppgaver/

Solution: 26.1:

See solution for exercise 3.a in this problem set. Insert numbers from exercise.

26.3:

$$H = \frac{1}{\mu_0} B - M$$

Answer: 26.1:

a) 3.77×10^{-2} T

b) 3.78×10^{-2} T