Fys2160 – 2013 – Oblig 9

Photon Gas and Ideal Gas Comparison

An $L \times L \times L$ cavity is in thermal equilbrium with a gas of photons inside the cavity at a temperature T.

(a) Show that the total number of photons in the gas is

$$N = 2.404\pi^{-2}V\left(\frac{kT}{\hbar c}\right)^3 . \tag{1}$$

(b) Show that the total energy of the photons in the gas is

$$U = aVT^4 , \tag{2}$$

where

$$a = \frac{8\pi^5}{15h^3c^3}k^4 . \tag{3}$$

(c) Show that the entropy of the photon gas is

$$S/N = 3.602k . \tag{4}$$

(d) Show that the pressure of the photo gas can be found from

$$p = -\left(\frac{\partial U}{\partial V}\right)_S . \tag{5}$$

(e) Show that the pressure of the photon gas is

$$p = \frac{1}{3}\frac{U}{V} . \tag{6}$$

(f) Find an expression for pV expressed in NkT for the photon gas.

(g) For an ideal gas, find the pressure as a function of the energy density.

(h) Assume we have an ideal hydrogen gass with density 1 mol cm^{-3}. At what temperature is the photon pressure equal to the ideal gas pressure?

Photon Gas Engine

In this project we will address a cyclic, reverible machine using a photo-gas as its medium. We assume that the photon gas can be modelled as the radiation in a cubic cavity of size $V = L^3$. The energy density of the gas is

$$\frac{U}{V} = aT^4 , \; a = \frac{\pi^2 k^4}{15\hbar^3 c^3} . \tag{7}$$

The machine consists of the following proesses: an isotherma process from 1 to 2, an isochor ($V = const.$) processe from 2 to 3, and an adiabatic expansion from 3 to 1. You can assume that the machine does positive work.

(a) Find the entropy, S, and the pressure p for the photon gas expressed in a, V, and T.

(b) Draw the cycle schematically in a p, V diagram and in an S-T diagram. Use the expression from above to draw the expressions realistically.

(c) Determine the efficieny of the machine. Expression your answer in V_1 and V_3.

End of Oblig 9