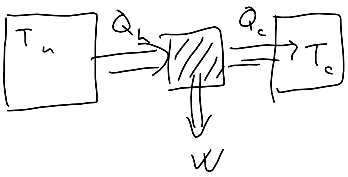
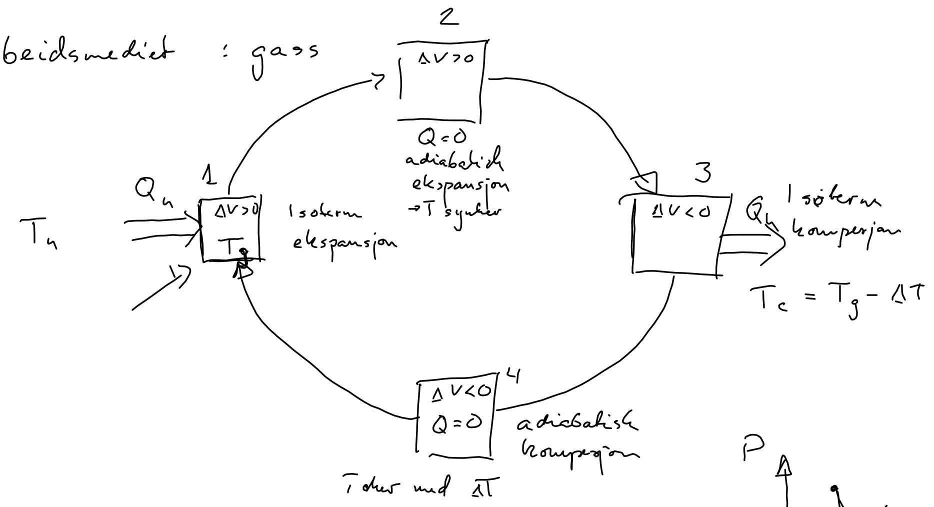
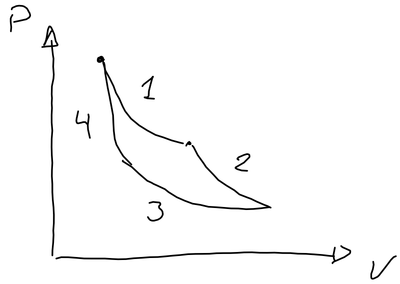


Arbeidsmediet : gass





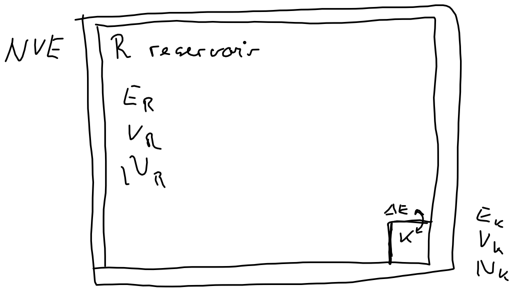
Ideell gass Isoterm eksp/komp : $P = \frac{\text{konst}}{V}$

Adiabatisk —— —— : $P = \frac{\text{konst}}{V^{7/5}}$

Carnot Ideell gass : maksimal effektivitet :

$$e = 1 - \frac{T_c}{T_h}$$

Til nå NVE. Konst temperatur $\Rightarrow$ NVT (Kanoniske system)



R - varmebad $\quad E_R, V_R N_R \gg E_K V_K N_K$

utveksling av ΔE $\Rightarrow$ påvirker ikke R

Likevekt $\quad T_K = T_R$

$\Rightarrow$ K: NVT

$$E = E_0 = E_R + E_K$$
$$V = V_R + V_K$$
$$N = N_R + N_K$$

Mikrotilstand i med $E_K = \varepsilon_i$ (mange mikrotilst. med ε_i) $\Rightarrow$ $\underline{\Omega_K = 1}$

$$E_R = E_0 - \varepsilon_i$$

Hva er sannsynligh. til tilstand i?

$$P(i) = \frac{\Omega_R \Omega_K}{\sum \Omega_R \Omega_K} = C \cdot \Omega_R(E_0 - \varepsilon_i)$$

$$\ln P(i) = \ln C + \ln \Omega_R(E_0 - \varepsilon_i)$$

$\varepsilon_i \ll E_0$

$$\ln P(i) = \underline{\ln C + \ln \Omega_R(E_0)} + \frac{\partial \ln \Omega_R(E)}{\partial E}(-\varepsilon_i) + \mathcal{O}(\varepsilon_i^2)$$

$S = k \ln \Omega$

$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V,N}$

$$= -\ln Z - \frac{\partial S(E)}{k\partial E} \cdot \varepsilon_i$$
$$= -\ln Z - \frac{\varepsilon_i}{kT}$$

$$\boxed{P(i) = \frac{1}{Z} e^{-\varepsilon_i/kT}}$$

$e^{-\varepsilon_i/kT}$ - Boltzmann-faktor

$$\sum_i P(i) = 1 = \frac{1}{Z}\sum_i e^{-\varepsilon_i/kT}$$

Tilstandssummen

Partisjonsfunksjonen

$$\boxed{Z = \sum_i e^{-\varepsilon_i/kT}}$$

$$\beta = 1/kT$$

Sannsynligheten for at et system i likevekt med gitt N, V, T er i tilstand i med energien ε_i:

$$P(i) = \frac{e^{-\beta\varepsilon_i}}{Z}$$

Tilstandssummen (part. funksj.) Z er summen over alle tilstander i systemet med N, V, T konst.

$$Z(N,V,T) = \sum_i e^{-\beta\varepsilon_i}$$
$$= \sum_{\varepsilon} g(\varepsilon_i)\, e^{-\beta\varepsilon_i}$$

der $g(\varepsilon_i)$ er antallet tilstander med energi ε_i, (degenerasjonsgraden)

Middelverdier $\bar{Q}_i = \sum_i Q_i P(i) = \frac{1}{Z}\sum_i Q_i e^{-\beta\varepsilon_i}$

Midlere energi: $\bar{E} = \frac{1}{Z}\sum_i \varepsilon_i e^{-\beta\varepsilon_i}$ $\qquad \frac{\partial e^{-\beta\varepsilon_i}}{\partial\beta} = -\varepsilon_i e^{-\beta\varepsilon_i}$

$$= -\frac{1}{Z}\frac{\partial}{\partial\beta}\sum_i e^{-\beta\varepsilon_i} = \boxed{-\frac{1}{Z}\frac{\partial Z}{\partial\beta}}$$

$$\boxed{\bar{E} = -\frac{\partial \ln Z}{\partial\beta}} \qquad \beta = \frac{1}{kT}$$

Varmekapasitet

$$C_V = \left(\frac{\partial\varepsilon}{\partial T}\right)_{N,V} = \frac{\partial\varepsilon}{\partial\beta}\frac{\partial\beta}{\partial T} = -\frac{\partial\varepsilon}{\partial\beta}\cdot\frac{1}{kT^2}$$

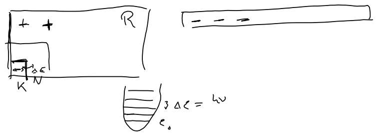
$$= \frac{1}{kT}\cdot\frac{\partial^2\ln Z}{\partial\beta^2} = \frac{1}{kT^2}\left[\frac{1}{Z}\frac{\partial^2 Z}{\partial\beta^2} - \frac{1}{Z^2}\left(\frac{\partial Z}{\partial\beta}\right)^2\right]$$

$$= \frac{1}{kT^2}\left[\overline{E^2} - \bar{E}^2\right] \quad \text{varians av energien}$$

Eksempel 1 Einsteinkrystallen

1 enkelt oscillator

$T_k = T_R$



$$\varepsilon_i = \varepsilon_0 + i\Delta\varepsilon$$

Tilst. sum $\quad Z = \sum_{i=0}^{N} e^{-\beta\varepsilon_i} = \sum_{i=0}^{N} e^{-i\beta\Delta\varepsilon}$

Geometrisk rekke $\quad a + ak + ak^2 \ldots ak^{n-1} = a\frac{k^n-1}{k-1} \qquad k = e^{-\beta\Delta\varepsilon}$

$$Z = \frac{e^{-\beta\Delta\varepsilon\cdot N}-1}{e^{-\beta\Delta\varepsilon}-1} \xrightarrow{N\to\infty} \boxed{\frac{1}{1-e^{-\beta\Delta\varepsilon}}}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial\beta} = -\frac{\partial}{\partial\beta}\ln Z$$

$$= -\frac{\partial}{\partial\beta}\ln\left(\frac{1}{1-e^{-\beta\Delta\varepsilon}}\right) = \frac{\partial}{\partial\beta}\ln(1-e^{-\beta\Delta\varepsilon})$$

$$= \frac{1}{1-e^{-\beta\Delta\varepsilon}}\frac{\partial(1-e^{-\beta\Delta\varepsilon})}{\partial\beta} = \frac{\Delta\varepsilon\, e^{-\beta\Delta\varepsilon}}{1-e^{-\beta\Delta\varepsilon}}$$

$$= \frac{\Delta\varepsilon}{e^{\beta\Delta\varepsilon}-1}$$

$$\bar{E}_N = N\bar{E}_1 = \frac{N\Delta\varepsilon}{e^{\beta\Delta\varepsilon}-1}$$

Paramagnet

N spinn $\frac{1}{2}$ - partikler i et konst. magnetfelt $\vec{B}$

$\vec{B}$ ↑ ↑↑↓↑↑↓ $\quad +\frac{1}{2}, -\frac{1}{2}\quad$ spinn/dipoler × vv. med feltet $\vec{B}$ × vv. ikke med hverandre

E: μB ↓, $-\mu B$ ↑

$$E = \mu B(N_\downarrow - N_\uparrow)$$

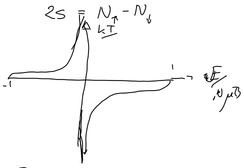
$$M = \mu(N_\uparrow - N_\downarrow) = -\frac{E}{B}$$

$$\Omega(N\uparrow) = \frac{N!}{N_\uparrow!\,N_\downarrow!}$$

Stirlings approks.

$$\Omega(N,s) = C e^{-\frac{2s^2}{N}} \qquad 2s = N_\uparrow - N_\downarrow$$

$$\frac{E}{N\mu B} = -\tanh\left(\frac{\mu B}{kT}\right)$$



Boltzmann-statistikk:

Én dipol $\quad Z = \sum_s e^{-\beta E(s)} = e^{\beta\mu B} + e^{-\beta\mu B}$

$$= 2\cosh(\beta\mu B)$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial\beta}$$

$$= \frac{-1}{2\cosh(\beta\mu B)}\cdot 2\sin(\beta\mu B)\cdot\mu B$$

$$\bar{E} = -\mu B\tanh(\beta\mu B)$$

N dipoler $\quad \bar{E}_n = -N\mu B\tanh(\beta\mu B)$