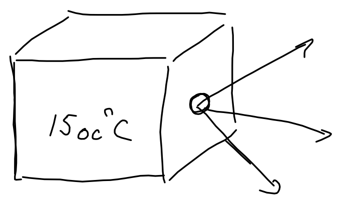


Stråling elektromagnetiske bølger

Maxwell 1865

Energitetthet $u(\nu, T)$

Strålingstrykket $P = \frac{u}{3}$

TD1 $\left(\frac{1}{dV}\right)_T$

$$dU = T\,dS - p\,dV$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p\left(\frac{\partial V}{\partial V}\right)_T \; 1$$

Maxwells relasjoner $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$$u = u\left(\frac{\partial V}{\partial V}\right) = \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P \qquad U = uV$$

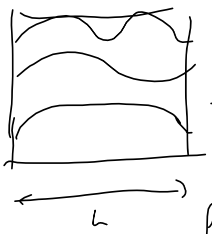
$$p = \frac{u}{3} \qquad u = \frac{T}{3}\left(\frac{\partial u}{\partial T}\right) - \frac{u}{3}$$

$$\frac{4u}{3} = \frac{T}{3}\frac{du}{dT} \Rightarrow \frac{dT}{T} = \frac{1}{4}\frac{du}{u}$$

$$\Rightarrow \underline{u} = \underline{\underline{a}} \cdot T^4$$

empirisk konst

Stefan Boltzmanns strålingslov



$\lambda_n = \frac{2L}{n}$

$f = \frac{nc}{2L} = n f_0$

$f_0 = \frac{c}{2L}$

partikler m/masse: $E_n = \frac{p^2}{2m} = \frac{h^2 n^2}{8mL^2}$

Planck EM-bølger $E_n = \frac{hc}{\lambda_n} = hnf_0 = \frac{hcn}{2L}$ $n = 0, 1, \ldots$

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n} = 1 + e^{-\beta h f_0} + e^{-2\beta h f_0} \ldots$$

$$= \frac{1}{1 - e^{-\beta h f_0}}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = \frac{hf_0}{e^{\beta h f_0} - 1} = hf_0 \cdot \bar{n}_{Pl}$$

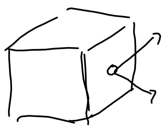
Planck-fordelingen $\bar{n}_{Pl} = \frac{1}{e^{\beta h f_0} - 1} = n(\beta \cdot \varepsilon)$ $\varepsilon|_T$ $T|_\varepsilon$ $\beta = \frac{1}{kT}$

__Fotoner er bosoner__

$$\bar{n}_{BE} = \bar{n}_{Pl}$$

$$\frac{1}{e^{(\varepsilon - \mu)\beta} - 1} = \frac{1}{e^{\beta\varepsilon} - 1} \quad \Big| \Rightarrow \mu = 0$$

Ant. fotoner ikke konservert.



Stråling ut, ikke inn __T=0__

Z, $\bar{E}$, $\bar{n}_{PL}$

U totalenergien

$u(\varepsilon, T)$ energitettheten

Totalenergien (generelt) $U = 2 \iiint \varepsilon(\vec{n})\, n(\varepsilon)\, g(\varepsilon)\, dn$

(2 ← fermioner: spinn; fotoner: polarisasjonsretn.) ($g(\varepsilon)$ ← 1 fotoner)

Sfæriske koordinater $U = 2 \cdot \frac{\pi}{2} \int_0^\infty n^2\, \varepsilon(n)\, \bar{n}_{PL}(\varepsilon)\, dn$

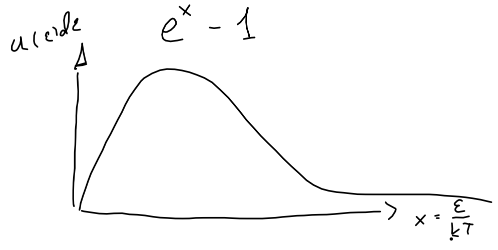
($\frac{\pi}{2} = \int_0^{\pi/2}\int_0^{\pi/2} \sin\theta\, d\theta\, d\varphi$)

$n \rightarrow \varepsilon$

$$L^3 \rightarrow \frac{U}{V} = \int_0^\infty \frac{8\pi\varepsilon^3}{(hc)^3} \frac{1}{e^{\beta\varepsilon} - 1}\, d\varepsilon = \int_0^\infty u(\varepsilon)\, d\varepsilon$$

Energispekteret : $u(\varepsilon)d\varepsilon = \frac{8\pi}{(hc)^3 \beta^4} \frac{x^3}{e^x - 1}\, dx$ $\boxed{x = \beta\varepsilon}$

$$\frac{x^3}{e^x - 1}$$



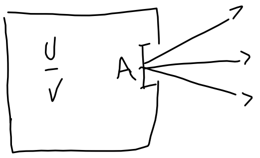
$\frac{1}{\beta^4} \sim T^4 \Rightarrow \frac{U}{V} = a \cdot T^4$

$\boxed{\varepsilon_n = hf \cdot n}$

$$\frac{U}{V} = \frac{8\pi^5 (kT)^4}{15\,(hc)^3} = aT^4$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 4aT^3$$

$$S(T) = \int_0^T \frac{C_V}{T'}\, dT' = \frac{4}{3} a T^3$$



$$\frac{\text{effekten}}{A} = \frac{\text{lengde}}{\text{tid}} \cdot \frac{\text{Energi}}{\text{Volum}}$$

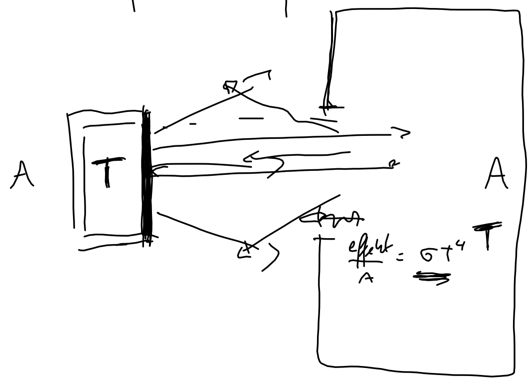
↑ c lyshastigheten

$$= c \cdot a T^4 \cdot \frac{1}{4}$$

$$= \sigma T^4 \quad , \quad \sigma = \frac{ca}{4}$$

Stefan-Boltzmannkonstanten $\sigma = 5{,}67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

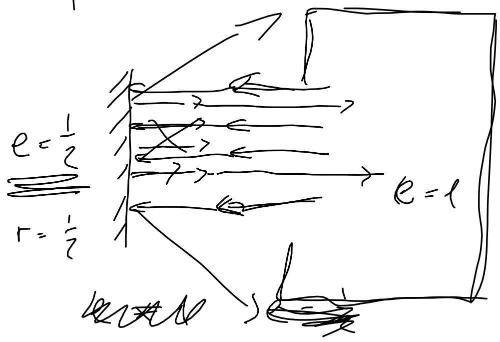
$$= \frac{2\pi^5 k^4}{15 \cdot h^3 c^2}$$



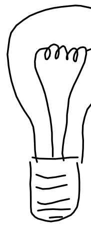
- samme areal
- samme temp
- samme fraksjon av strålingen treffer den andre strålingskilden
- samme emissivitet. $e = 1$

Hvis strålt effekten fra flaten ≠ strålte effekten fra hulrommet

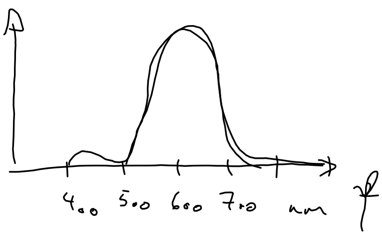
⇒ En flate med emissivitet $e = 1$ stråler som et svart hulrom.



$$\text{effekt} = \sigma \cdot e \cdot A \cdot T^4$$



vs



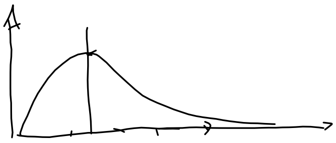
~ 100% av energien er i det synlige området

100 W vs 30 W ?

Utstrålt effekt $= \sigma e A T^4$

$$A = \frac{\text{effekt}}{\sigma e T^4} = \frac{100\,W}{5{,}67\cdot 10^{-8}\,\frac{W}{m^2K^4}\cdot\frac{1}{3}(3000\,K)^4}$$

$$= 66\ mm^2$$

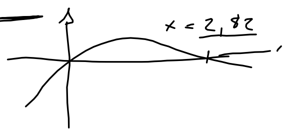


Hvilken fotonenergi har størst energitetthet ?

$$u(\beta,\varepsilon) = \frac{8\pi}{(hc)^3\beta^4}\frac{x^3}{e^x-1}$$

$x = \beta\varepsilon$

$$\frac{\partial u}{\partial x} = \frac{(e^x(x-3)+3)x^2}{(e^x-1)^2} = 0$$



$$\varepsilon = \frac{x}{\beta} = kTx = 0{,}26\cdot 2{,}82\ eV$$

$$= 0{,}73\ eV$$

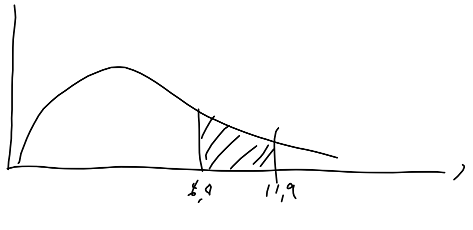
$$kT = 8{,}6\cdot 10^{-5}\,\frac{eV}{K}\cdot 3000\,K = 0{,}26\ eV$$

$$\lambda = \frac{hc}{\varepsilon} = \frac{1230\ eVnm}{0{,}73\ eV} = 1700\ nm$$

Synlige spekter λ: 400 nm $\Rightarrow$ $\varepsilon = \frac{hc}{\lambda} = \frac{1230}{400}\ eV = 3{,}1\ eV$ $\quad x = 11{,}9$

700 nm $\quad \varepsilon = 1{,}7\ eV \quad x = \beta\varepsilon = 6{,}8$

Hvor mye av energien til lyspæren er i det synlige spekteret ?



$$\frac{\int_{6,8}^{11,9}\frac{x^3}{e^x-1}\,dx}{\int_0^{\infty}\frac{x^3}{e^x-1}\,dx}$$