Solutions to exercises week 38

FYS2160

Kristian Bjørke, Knut Oddvar Høie Vadla

November 7, 2017

Schroeder 2.30

a)

$\Omega_{\text{total}} = \frac{2^{4N}}{\sqrt{8\pi N}}$

$\frac{S}{k} = \ln \Omega_{\text{total}} = \ln \left[\frac{2^{4N}}{\sqrt{8\pi N}} \right] = 4N \cdot \ln 2 - \ln \sqrt{8\pi N}$

N $= 10^{23} \Rightarrow \frac{S}{k} = 2.77 \cdot 10^{23} - 28.1$

b)

$\Omega_{\text{most likely}} = \frac{2^{4N}}{4\pi N}$

$\frac{S}{k} = \ln \Omega_{\text{most likely}} = \ln \left[\frac{2^{4N}}{4\pi N} \right] = 4N \cdot \ln 2 - \ln 4\pi N$

N $= 10^{23} \Rightarrow \frac{S}{k} = 2.77 \cdot 10^{23} - 55.5$

c)

Small difference in entropy, $\Delta S/k = 55.5 - 28.1 = 27.4$, time scale irrelevant.

d)

Entropy decrease by 27 units out of 2.8×10^{23}, insignificant.

Schroeder 2.37

$N_A = (1-x)N,\ N_B = xN$

$V_i^A = (1-x)V_f$, $V_i^B = xV_f$

Use $\Delta S_X = -N_X k \ln \frac{V_i^X}{V_f}$ to get:

$$\begin{aligned}\Delta S_{\text{mixing}} &= \Delta S_A + \Delta S_B \\ &= -(1-x)Nk\ln(1+x) - xNk\ln x \\ &= -Nk\left[x\ln x + (1-x)\ln(1-x)\right]\end{aligned}$$

For $x = \frac{1}{2}$:

$\Delta S_{\text{mixing}} = -Nk\left[\ln\frac{1}{2}\right] = Nk\ln 2$

(N is total number of molecules, which is $2N$ in eq. 2.54)

Schroeder 2.38

When we allow the system to mix, assuming an ideal mixture, the only changes will be that the different molecules can change places with each other. This gives:

$\Delta\Omega = \binom{N}{N_A} = \frac{N!}{N_A!N_B!}$

$\Delta S_{\text{mixing}} = k\ln\binom{N}{N_A} = k\ln\frac{N!}{N_A!N_B!}$

Use Stirling's approximation ($\ln N! \approx N\ln N - N$) to get:

$\Delta S_{\text{mixing}} = -Nk\left[x\ln x + (1-x)\ln(1-x)\right]$

Schroeder 3.3

Initially: $\frac{\delta S_A}{\delta U_A} > \frac{\delta S_B}{\delta U_B}$

U_A will increase, U_B will decrease until:

$\frac{\delta S_A}{\delta U_A} = \frac{\delta S_B}{\delta U_B}$

Schroeder 3.38

From ideal gas law:

$P_i = x_i P = x_i NkT/V = N_i kT/V$

, so if P_i is fixed the gas component i is unchanged when adding another component to the mix.

For a mix of two ideal gases total entropy given by:

$S_{\text{total}} = S_A(U_A, V, N_A) + S_B(U_B, V, N_B)$

Chemical potential for gas A is:

$\mu_A = -T\left(\frac{\delta S}{\delta N_A}\right)_{U,V,N_B} = -T\left(\frac{\delta S_A}{\delta N_A}\right)_{U,V}$

, same as if gas B was not present.

Schroeder 4.3

a)

$Q_c = W\left(\frac{1}{e} - 1\right) = 1.5GW$

b)

Every second 1.5×10^9 J dumped in 10^5 kg water.

Heat capacity of water: $C = 4186$ J/°C.

$\Delta T = \frac{Q}{C} = \frac{15\text{kJ}}{4.2\text{kJ/°C}} = 3.6$ °C

c)

Latent heat at room temp: $L = 2.4$ kJ/g

Evaporation rate: $\frac{Q}{L} = 600$ kg/s $= 0.6$ m^2/s, 0.6% of river

Schroeder 4.7

Need to dump heat into “cold” reservoir separate from room to be effective, otherwise it would raise temperature not lower it.

Schroeder 4.8

Q_h always bigger than Q_c, so temperature in room will increase.

Schroeder 4.14

a)

COP is defined as benefit divided by cost, heat pump aims to heat up so:

$COP = \frac{\text{benefit}}{\text{cost}} = \frac{Q_h}{W}$

b)

$Q_h = Q_c + W$

$COP = \frac{1}{1 - Q_c/Q_h} > 1$

c)

2. law of thermodynamics:

$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c}$

$COP \geq \frac{T_h}{T_h - T_c}$

d)

Electric heater: $Q_h = W$, $COP = 1$

Heat pump: $COP > 1$

For $T_h = 25°\text{C}$ and $T_c = 0°\text{C}$ COP can in principle be as high as 12 for heat pump (usually much lower but still larger than 1).

Compendium 6.1

a)

Z_4	$\{X_1, X_2, X_3, X_4\}$
4	$\{+,+,+,+\}$
2	$\{+,+,+,-\}$
	$\{+,+,-,+\}$
	$\{+,-,+,+\}$
	$\{-,+,+,+\}$
0	$\{+,+,-,-\}$
	$\{+,-,+,-\}$
	$\{-,+,+,-\}$
	$\{+,-,-,+\}$
	$\{-,+,-,+\}$
	$\{-,-,+,+\}$
-2	$\{+,-,-,-\}$
	$\{-,+,-,-\}$
	$\{-,-,+,-\}$
	$\{-,-,-,+\}$
-4	$\{-,-,-,-\}$

b)

$P(Z_4 = 4) = \frac{1}{16}$, $P(Z_4 = 2) = \frac{4}{16}$, $P(Z_4 = 0) = \frac{6}{16}$

$P(Z_4 = -2) = \frac{4}{16}$, $P(Z_4 = -4) = \frac{1}{16}$

b)

$\Omega(Z_4 = 4) = \binom{4}{0} = 1$, $\Omega(Z_4 = 2) = \binom{4}{1} = 4$, $\Omega(Z_4 = 0) = \binom{4}{2} = 6$

$\Omega(Z_4 = -2) = \binom{4}{3} = 4$, $\Omega(Z_4 = -4) = \binom{4}{4} = 1$,