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# Electron gas

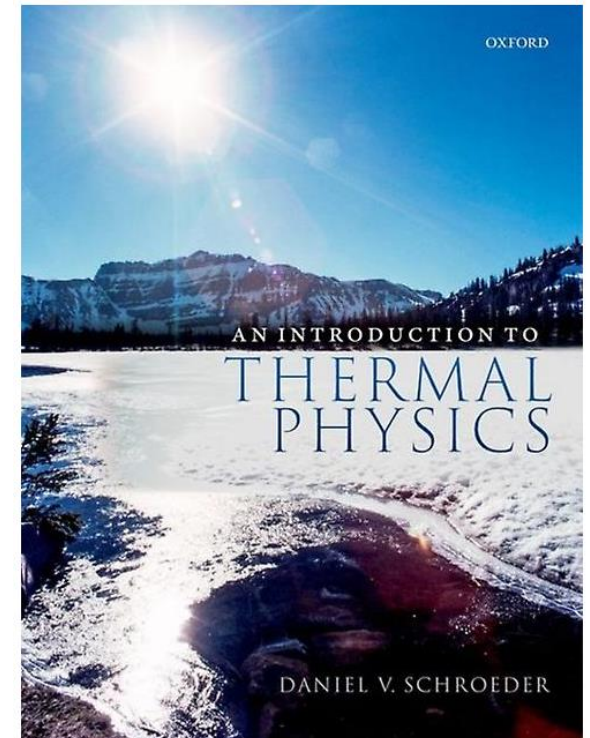
6 Boltzmann Statistics

7 Quantum Statistics

7.1 The Gibbs Factor

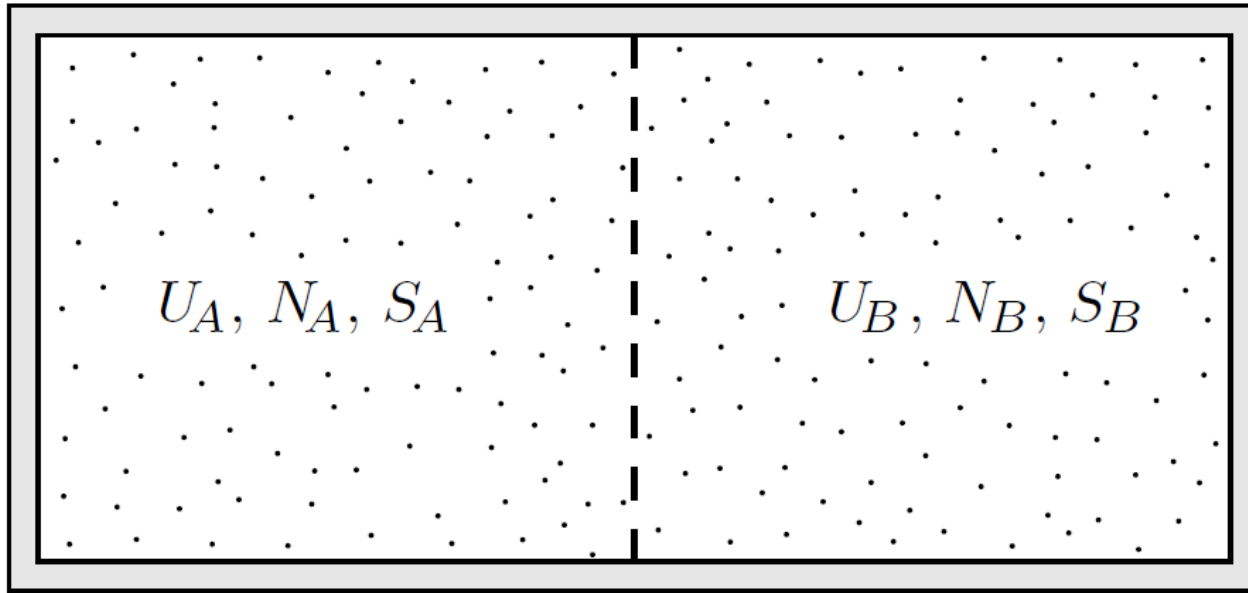
7.2 Bosons and Fermions

7.3 Degenerate Fermi Gases



# Diffusive equilibrium and **chemical potential**

When two systems are in thermal equilibrium, their **temperatures** are the same. When they're in mechanical equilibrium, their **pressures** are the same. **What quantity is the same when they're in diffusive equilibrium?**



DVS

An example of **two systems that can exchange both energy and particles** with total energy and number of particles fixed. **At equilibrium, the total entropy is at a maximum.**

$$\left(\frac{\partial S_{total}}{\partial U_A}\right)_{N_A, N_B} = 0 \quad \left(\frac{\partial S_{total}}{\partial N_A}\right)_{U_A, U_B} = 0$$

$$\frac{\partial S_{total}}{\partial U_A} = \frac{\partial S_A}{\partial U_A} + \frac{\partial S_B}{\partial U_A} = 0$$

$$\frac{\partial S_A}{\partial U_A} = -\frac{\partial S_B}{\partial U_A} = \frac{\partial S_B}{\partial U_B} \quad \left(\frac{\partial S}{\partial U}\right)_{N, V} \equiv \frac{1}{T}$$

At equilibrium:

$$\frac{\partial S_A}{\partial U_A} = \frac{\partial S_B}{\partial U_B}$$



Constant T

$$\frac{\partial S_A}{\partial N_A} = \frac{\partial S_B}{\partial N_B}$$



What parameter is constant here?

- **Chemical potential** is a key for understanding behavior of **electron gas**.

# Diffusive equilibrium and **chemical potential**

$$\frac{\partial S_A}{\partial N_A} = \frac{\partial S_B}{\partial N_B} \qquad -T \frac{\partial S_A}{\partial N_A} = -T \frac{\partial S_B}{\partial N_B}$$

$$\mu \equiv -T \left( \frac{\partial S}{\partial N} \right)_{U,V}$$

$\mu$  is **chemical potential**. In diffusive equilibrium,  $\mu_A = \mu_B$ .

$\mu$  is a parameter which is constant in diffusive equilibrium.

If two systems are not in diffusive equilibrium, then the **one with the larger value of  $\partial S/\partial N$  will tend to gain particles, since it will thereby gain more entropy than the other loses**. Because of the minus sign, this system has the smaller value of  $\mu$ . Therefore, **particles tend to flow from the system with higher  $\mu$  into the system with lower  $\mu$** .

# Types of interactions

Type of interaction	Exchanged quantity	Governing variable	Formula
thermal	energy	temperature	$\frac{1}{T} = \left( \frac{\partial S}{\partial U} \right)_{V,N}$
mechanical	volume	pressure	$\frac{P}{T} = \left( \frac{\partial S}{\partial V} \right)_{U,N}$
diffusive	particles	chemical potential	$\frac{\mu}{T} = - \left( \frac{\partial S}{\partial N} \right)_{U,V}$

# Generalized thermodynamic identity

Let us change  $U$  by  $dU$ ,  $V$  by  $dV$ , and  $N$  by  $dN$   
and see what will happen with  $S$ :

$$dS = \left(\frac{\partial S}{\partial U}\right)_{V,N} dU + \left(\frac{\partial S}{\partial V}\right)_{U,N} dV + \left(\frac{\partial S}{\partial N}\right)_{V,U} dN \quad \left(\frac{\partial S}{\partial U}\right)_{N,V} \equiv \frac{1}{T} \quad \mu \equiv -T \left(\frac{\partial S}{\partial N}\right)_{U,V}$$

$$dU = TdS - PdV$$

$$dS = \frac{dU}{T} + \frac{PdV}{T} - \frac{\mu dN}{T}$$

$$TdS = dU + PdV - \mu dN$$

$$dU = TdS - PdV + \mu dN$$

# Link to Gibbs free energy

$$dU = TdS - PdV + \mu dN$$

Generalized thermodynamic identity

$$dG = dU - TdS + PdV = \mu dN \quad \mu = \left( \frac{\partial G}{\partial N} \right)_{T,P}$$

# Chemical potential: different formulas

$$dU = TdS - PdV + \mu dN$$

Fixed  $U$  and  $V$ :

$$\mu = -T \left( \frac{\partial S}{\partial N} \right)_{U,V}$$

Fixed  $U$  and  $S$ :

$$\mu = P \left( \frac{\partial V}{\partial N} \right)_{U,S}$$

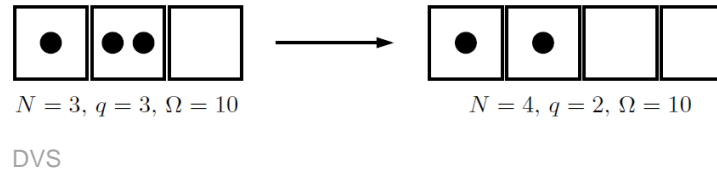
Fixed  $V$  and  $S$ :

$$\mu = \left( \frac{\partial U}{\partial N} \right)_{S,V}$$

Chemical potential is the amount by which a system's energy changes when one adds one particle and keeps the entropy and volume fixed.  $\mu$  has units of energy.

# Chemical potential: an example

$$\mu = \left( \frac{\partial U}{\partial N} \right)_{S,V}$$



Small **Einstein solid** with three oscillators and three units of energy  $\mathcal{E}$ , adding 4<sup>th</sup> oscillator:  $k \ln 10 \rightarrow k \ln 20$ . One unit of energy is necessary to remove  $\mu = -\epsilon / 1 = -\epsilon$ .



# Chemical potential of ideal gas

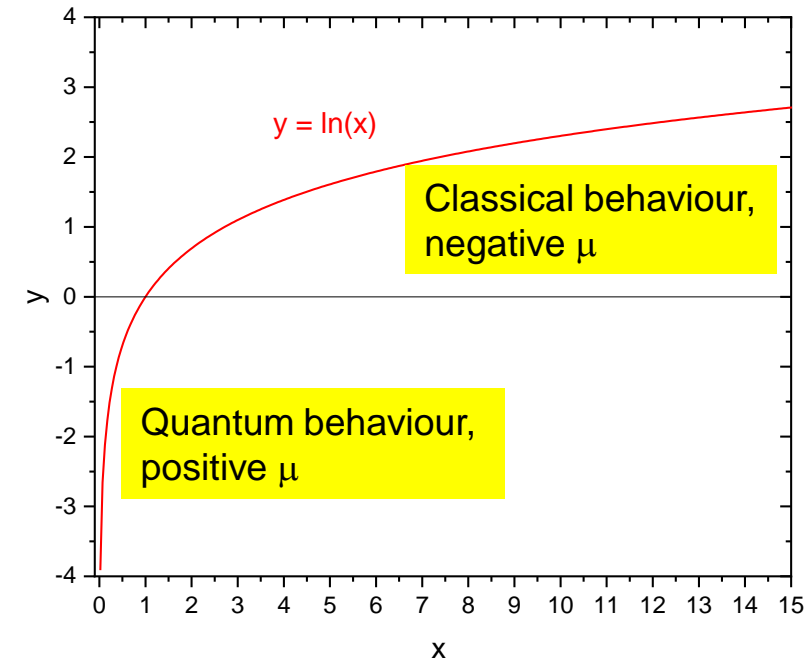
Sackur-Tetrode equation: 
$$S = Nk \left[ \ln \left( \frac{V}{N} \left( \frac{4\pi mU}{3Nh^2} \right)^{3/2} \right) + \frac{5}{2} \right]$$

$$\mu \equiv -T \left( \frac{\partial S}{\partial N} \right)_{U,V}$$

$$\mu = -kT \ln \left[ \frac{V}{N} \left( \frac{2\pi mkT}{h^2} \right)^{3/2} \right].$$

$$\mu = -kT \ln \left[ \frac{V}{N\vartheta_Q} \right]$$

$$\vartheta_Q = l_Q^3 = \left( \frac{h}{\sqrt{2\pi mkT}} \right)^3$$



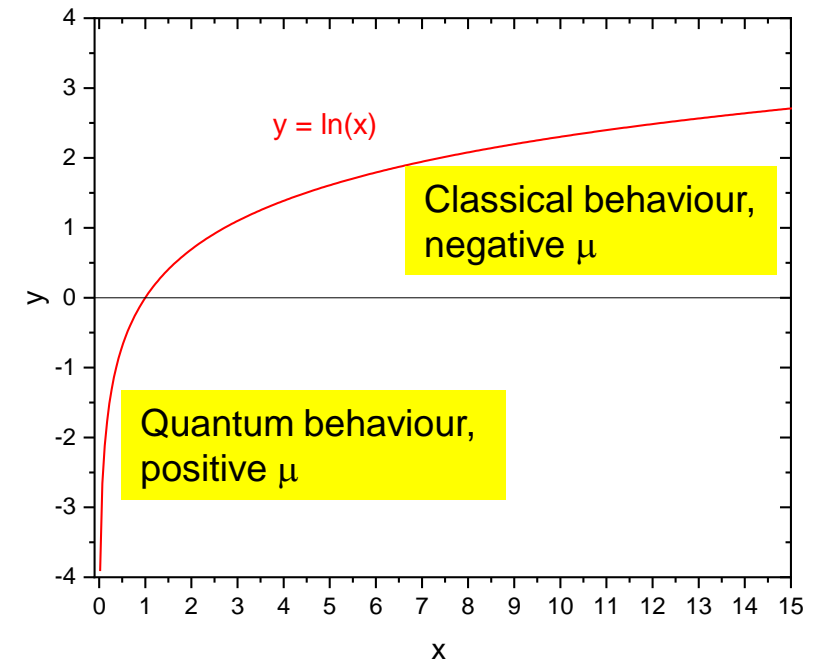
- The **sign of chemical potential depends on the ratio of the volume per one particle and the quantum volume  $\vartheta_Q$** .  $\mu$  is **negative** for  $\frac{V}{N} > \vartheta_Q$ , or for a non-dense system. A large mass of particles or large  $T$  results in a small  $\vartheta_Q$ .
- Reduction in mass and decrease in temperature results in  $\frac{V}{\vartheta_Q N} < 1$  and **positive  $\mu$** .

# Quantum volume and length

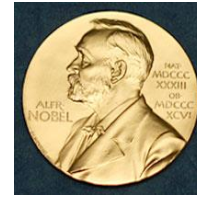
$$\mu = -kT \ln \left[ \frac{V}{N} \left( \frac{2\pi m kT}{h^2} \right)^{3/2} \right]. \quad \vartheta_Q = l_Q^3 = \left( \frac{h}{\sqrt{2\pi m kT}} \right)^3$$

$$\frac{h}{\sqrt{2\pi m kT}} \xrightarrow{\epsilon = \frac{p^2}{2m}} \frac{h}{\sqrt{2\pi m \epsilon}} \xrightarrow{p = \frac{\hbar h}{2\pi}} \frac{h}{p\sqrt{\pi}} \xrightarrow{\frac{2\pi}{\hbar} = \lambda_{dB}} \frac{2\pi}{\hbar\sqrt{\pi}} \xrightarrow{\lambda_{dB}} \frac{\lambda_{dB}}{\sqrt{\pi}}$$

- For the air we breathe, the average distance between molecules is about 3 nm while the **average de Broglie wavelength** is less than 0.02 nm, so condition  $\frac{V}{\vartheta_Q N} \gg 1$  is satisfied.
- For an electron at room temperature, **because of low mass**, the quantum volume is  $\vartheta_Q = (4.3 \text{ nm})^3$ , while the volume per conduction electron is roughly the volume of an atom,  $(0.2 \text{ nm})^3$ . Therefore, **electron gas** in metals at ambient conditions is **quantum gas** with  $\frac{V}{\vartheta_Q N} \ll 1$ .



# Electron gas



The Nobel Prize in Physics 1906

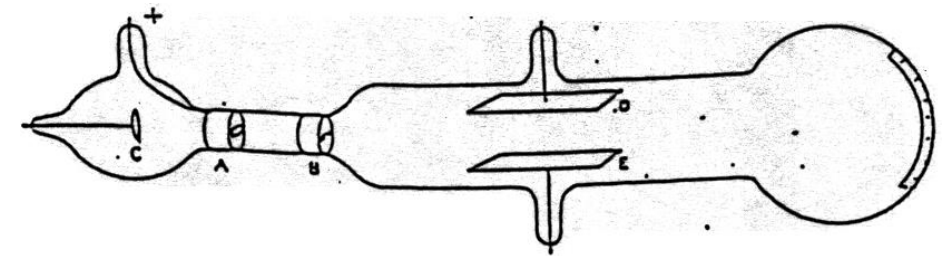
**Joseph John Thomson**

"in recognition of the great merits of his theoretical and experimental investigations on the **conduction of electricity** by gases"

'By carefully measuring how the cathode rays were deflected by electric and magnetic fields, Thomson was able to determine **the ratio between the electric charge (e) and the mass (m) of the rays**. Thomson's result was  $e/m = 1.8 \cdot 10^{11}$  coulombs/kg.

**The particle that J.J. Thomson discovered in 1897**, the electron, is a constituent of all the matter we are surrounded by. All atoms are made of a nucleus and electrons. **He received the Nobel Prize in 1906 for the discovery of the electron, the first elementary particle.'**

<http://www.nobelprize.org/educational/physics/vacuum/experiment-1.html>



[https://en.wikipedia.org/wiki/J.\\_J.\\_Thomson](https://en.wikipedia.org/wiki/J._J._Thomson)



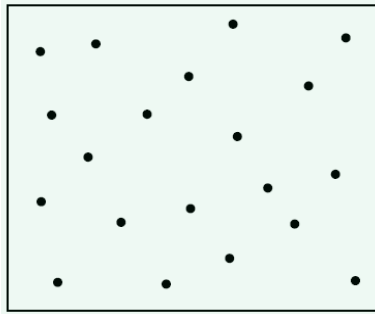
# Cavendish Laboratory



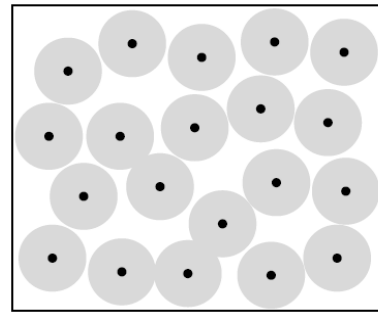


# Electron gas in vacuum

[https://en.wikipedia.org/wiki/Vacuum\\_tube](https://en.wikipedia.org/wiki/Vacuum_tube)



Normal gas



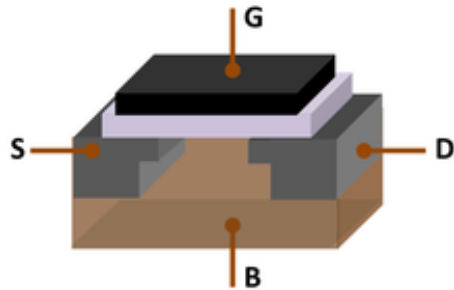
DVS

Quantum  
electron gas

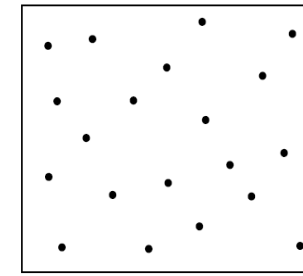


“The simplest vacuum tube, the **diode** (i.e. **Fleming valve**), invented in 1904 by **John Ambrose Fleming**, contains only a heated **electron-emitting cathode** and an anode. Electrons can only flow in one direction through the device—from the cathode to the anode. Adding one or more **control grids** within the tube allows the **current between the cathode and anode to be controlled by the voltage on the grids.**”

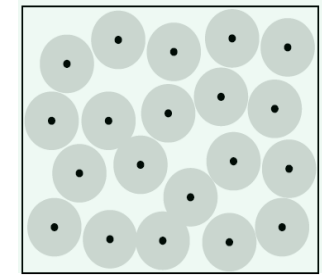
# Solid-state transistors



## Si-MOSFET



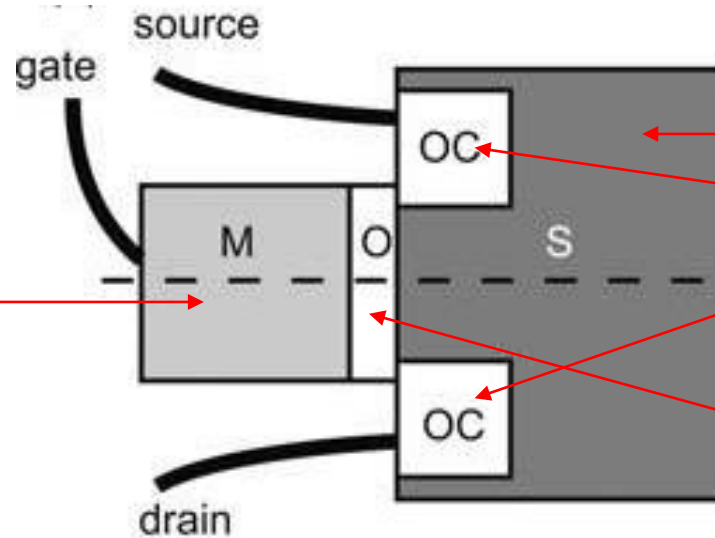
Normal gas



Quantum electron gas

DVS

Metallic gate

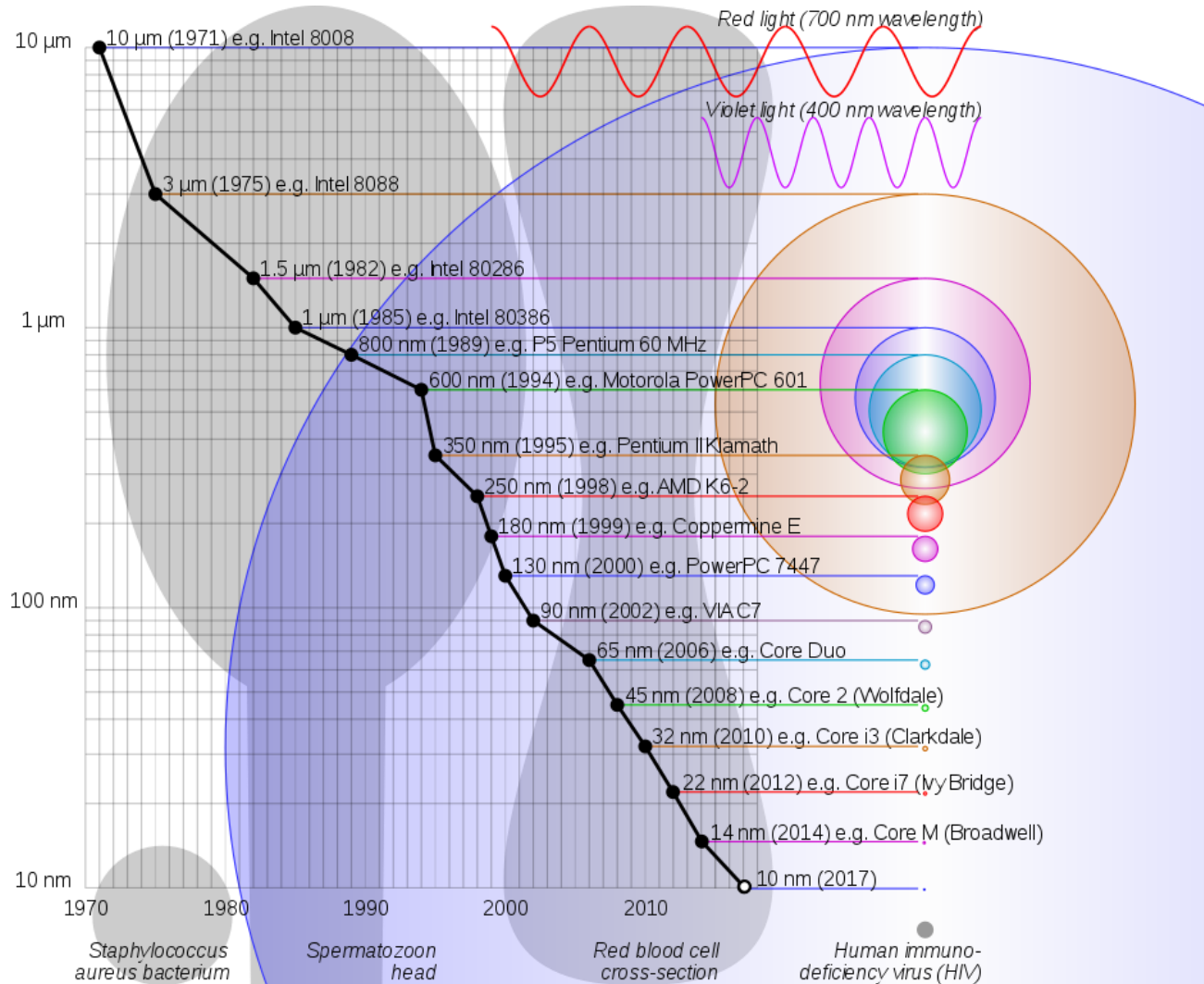


p-doped Si

Ohmic contacts, n-doped

Oxide, SiO<sub>2</sub>

# Progress in miniaturisation



**Semiconductor manufacturing processes**



- 10 µm – 1971
- 6 µm – 1974
- 3 µm – 1977
- 1.5 µm – 1982
- 1 µm – 1985
- 800 nm – 1989
- 600 nm – 1994
- 350 nm – 1995
- 250 nm – 1997
- 180 nm – 1999
- 130 nm – 2001
- 90 nm – 2004
- 65 nm – 2006
- 45 nm – 2008
- 32 nm – 2010
- 22 nm – 2012
- 14 nm – 2014
- 10 nm – 2017
- 7 nm – ~2018
- 5 nm – ~2020

Half-nodes

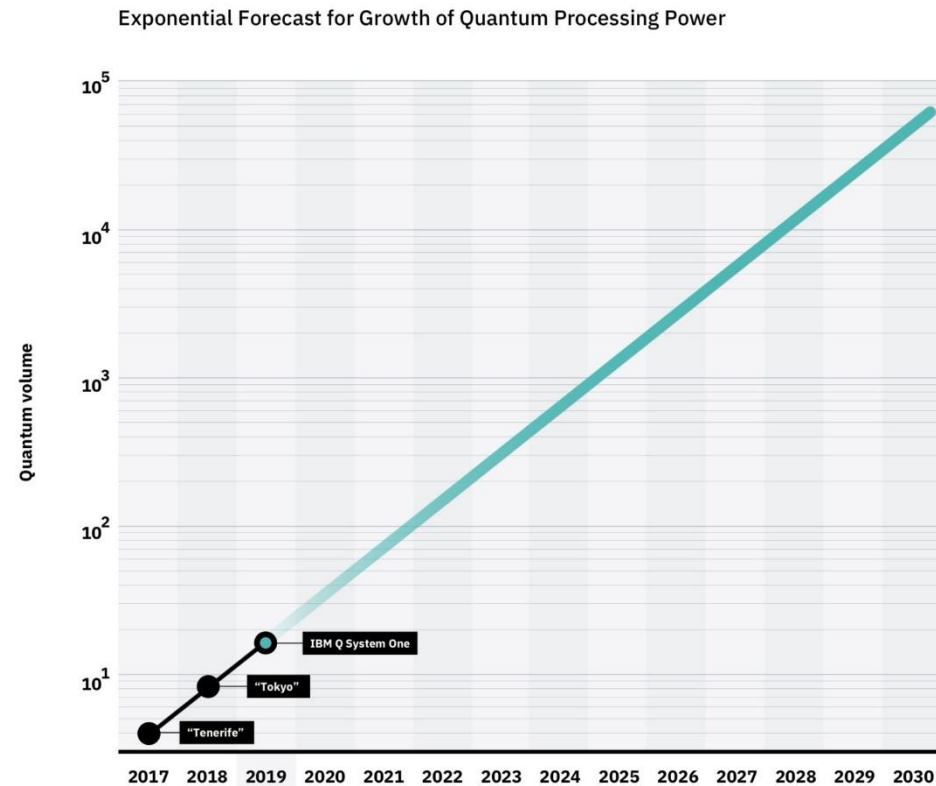
V · T · E





# Moor's law 2.0

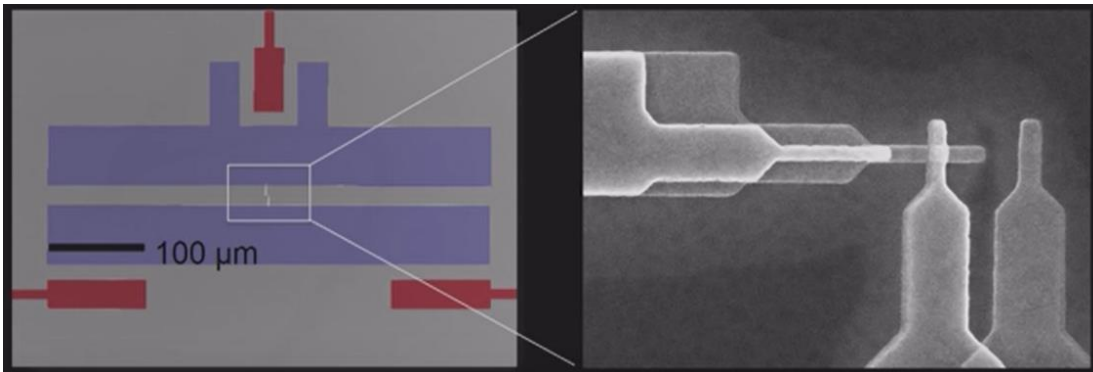
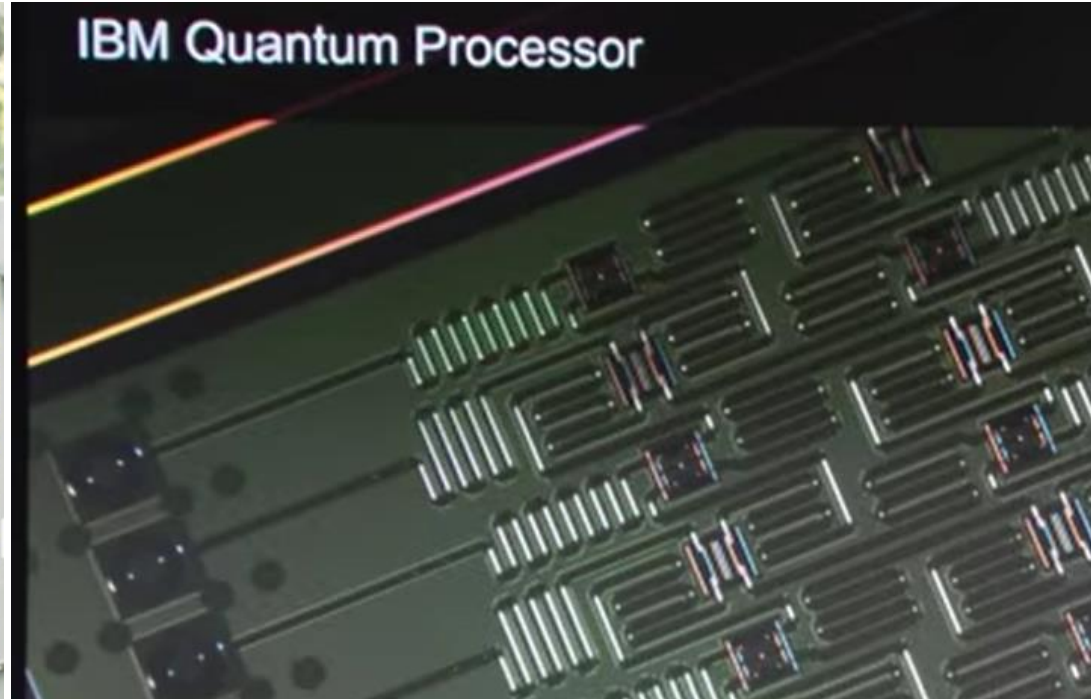
BOSTON, March 4, 2019 /PRNewswire/ -- At the 2019 [American Physical Society March Meeting](#), IBM (NYSE: [IBM](#)) unveiled a new scientific milestone, announcing its highest quantum volume to date.



IBM has doubled the power of its quantum computers annually since 2017.

[https://newsroom.ibm.com/2019-03-04-IBM-Achieves-Highest-Quantum-Volume-to-Date-Establishes-Roadmap-for-Reaching-Quantum-Advantage#assets\\_all](https://newsroom.ibm.com/2019-03-04-IBM-Achieves-Highest-Quantum-Volume-to-Date-Establishes-Roadmap-for-Reaching-Quantum-Advantage#assets_all)

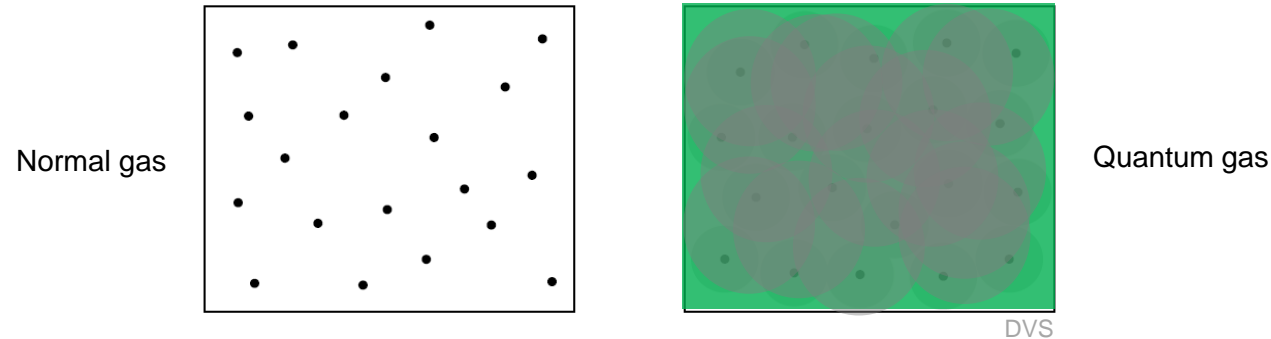
# Superconductivity and quantum computing



<https://techcrunch.com/2017/11/10/ibm-passes-major-milestone-with-20-and-50-qubit-quantum-computers-as-a-service/>

<https://www.youtube.com/watch?v=yy6TV9Dntlw>

# Bosons and fermions



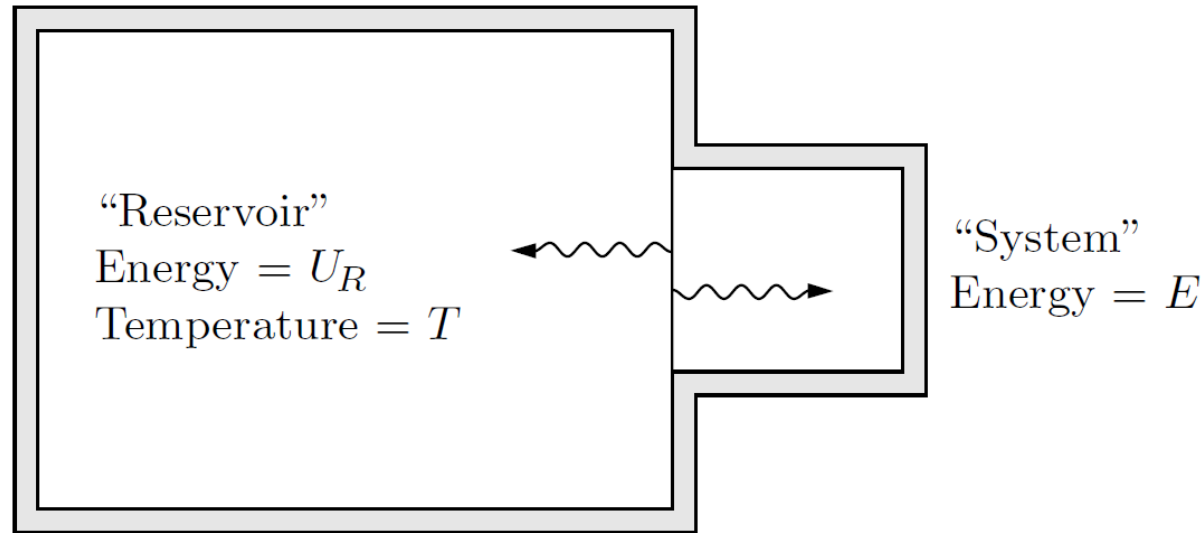
- For a dense system, **particles that try to occupy the same state can be divided in two groups.**
- Particles that **can share a state** with another are called **bosons**. Examples: photons and helium-4 atoms.
- Particles that **cannot share a state** with another are called **fermions**. Examples: electrons, protons, neutrons and helium-3 atoms.
- Particles with **integer spin** (0, 1, 2, etc., in units of  $h/2\pi$ ) are **bosons**.
- Particles with **half-integer spin** (1/2, 3/2, etc.) are **fermions**.

Is electron gas composed of fermions or bosons?

# Microcanonical, canonical, and **grand canonical** ensembles

In isolated systems or **microcanonical** ensemble, all allowed microstates had the same probability, which is “trivial” probability distribution. In **canonical** ensemble, members are assigned to states according to the Boltzmann probability distribution. It considers system in thermal contact with a much larger “reservoir” at some well-defined temperature allowing exchange of energy. **Grand canonical** ensemble allows exchange of particles too.

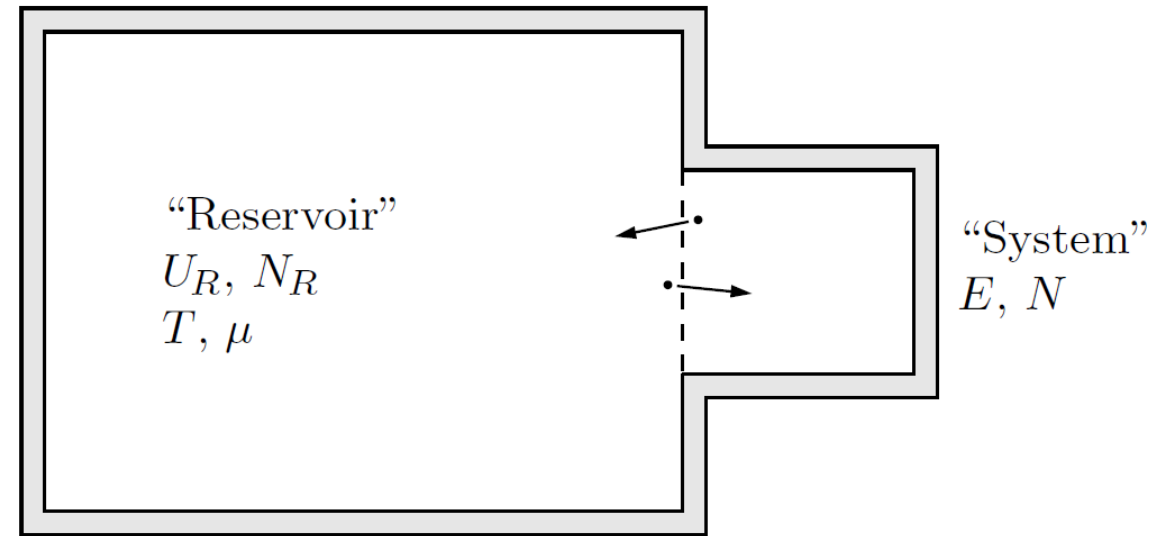
## Boltzmann and Maxwell distributions



DVS

Canonical ensemble

## Fermi-Dirac and Bose-Einstein distributions



DVS

Grand canonical ensemble

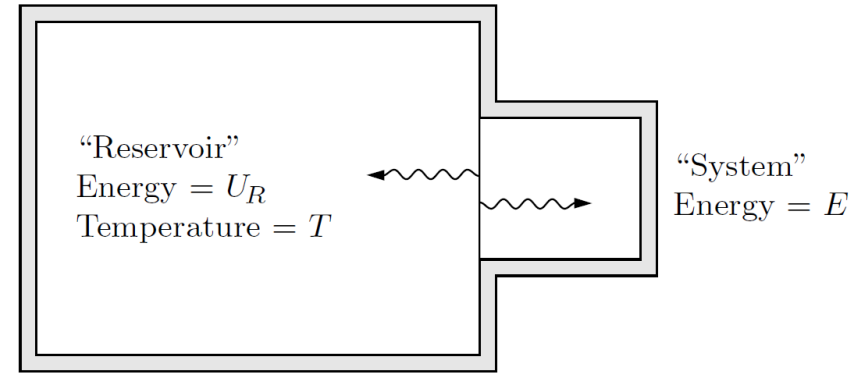
- **Chemical potential** is a key for understanding behavior of electron gas.

# Boltzmann statistics

Boltzmann statistics calculates probability of the system in the contact with reservoir having energy  $E$ . This probability is proportional to **multiplicity of reservoir**:

$$P(E) = C\Omega_R(E)$$

$$\Omega_R(E) = A\Omega_R(0) \quad S_R(E) = k \ln\Omega_R(0) + k \ln A$$



$$\Delta S_R = k \ln A \quad \Delta U = T\Delta S - P\Delta V + \mu\Delta N$$

$$E = -\Delta U_R = -T\Delta S_R \quad \Delta S_R = -\frac{E}{T}$$

$$A = e^{-E/kT}$$

$$P(E) = AC\Omega_R(0)$$

$$P(s) = \frac{1}{Z} e^{-\frac{E(s)}{kT}}$$

$$Z = \sum_s e^{-\frac{E(s)}{kT}}$$

Boltzmann distribution

$A$  is Boltzmann factor  $e^{-\frac{E}{kT}}$

$$P(E) = e^{-E/kT} C\Omega_R(0) = \frac{1}{Z} e^{-E/kT}$$

# Transition to Gibbs statistics

Boltzmann statistics calculates probability of the system in the contact with reservoir having energy  $E$ . This probability is proportional to multiplicity of reservoir:

$$P(E) = C\Omega_R(E)$$

$$\Omega(E) = A\Omega_R(0) \quad S_R(E) = k \ln\Omega_R(0) + k \ln A$$

$$\Delta S_R = k \ln A \quad \Delta U = T\Delta S - P\Delta V + \mu\Delta N$$

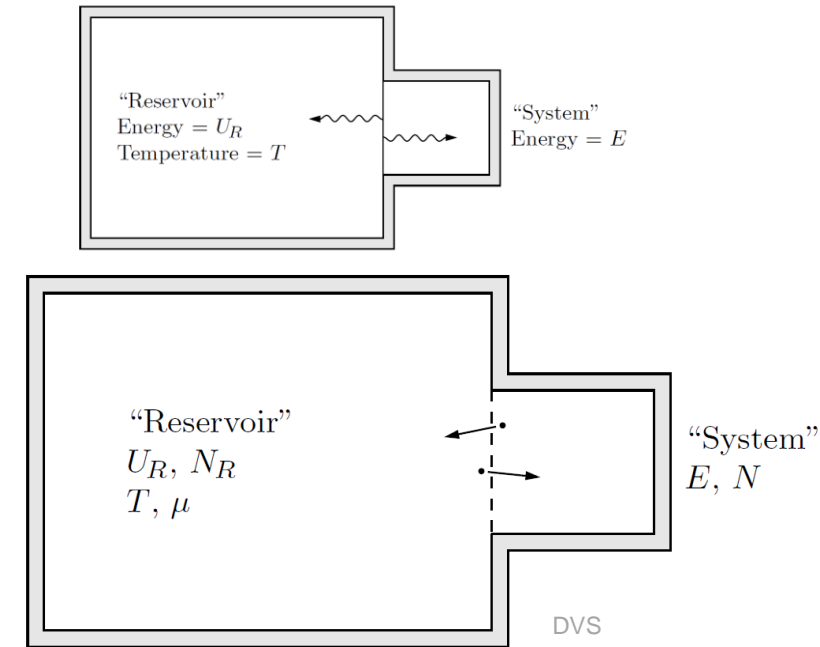
$$E = -\Delta U_R = -T\Delta S_R - \mu\Delta N_R \quad \Delta S_R = -\frac{E - \mu N}{T}$$

$$A = e^{-(E - \mu N)/kT}$$

$$\mathcal{P}(E) = A C \Omega_R(0)$$

A is Gibbs factor  $e^{-\frac{E - \mu N}{kT}}$

$$\mathcal{P}(E) = e^{-(E - \mu N)/kT} C \Omega_R(0) = \frac{1}{Z} e^{-(E - \mu N)/kT}$$



Gibbs distribution



# Fermi-Dirac distribution

Main idea is to consider a system as combination of **states for single-particles** and find average number of particles in these states. The **energy** when a state is occupied by a single particle is  $\epsilon$ . When the state is unoccupied, its energy is 0. If it is **occupied by  $n$  particles**, the energy is  $n\epsilon$ . The probability of the state being occupied by  $n$  particles is:

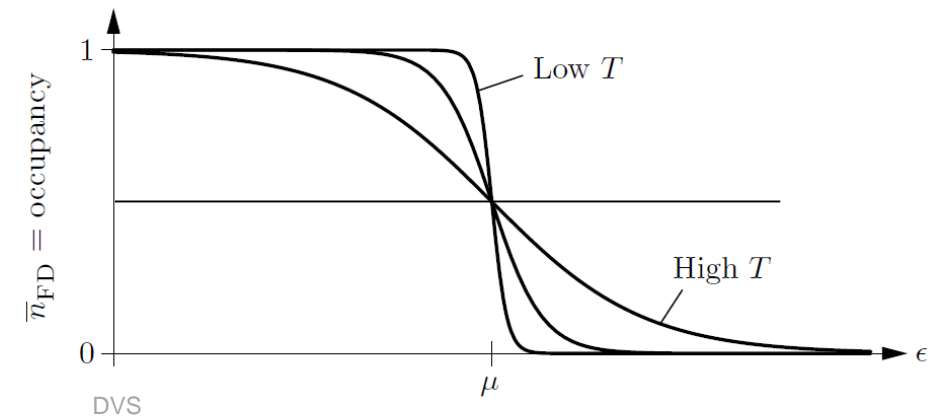
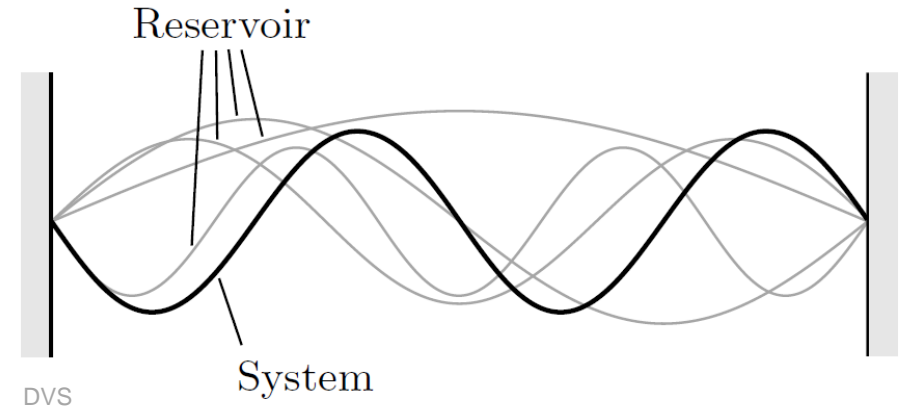
$$\mathcal{P}(n) = \frac{1}{\mathcal{Z}} e^{-\frac{n\epsilon - \mu n}{kT}} = \frac{1}{\mathcal{Z}} e^{-\frac{n(\epsilon - \mu)}{kT}}$$

If the particles are fermions, then  $n$  can only be 0 or 1, so the grand partition function is:  $\mathcal{Z} = 1 + e^{-\frac{\epsilon - \mu}{kT}}$ .

The average number of particles in the state or the occupancy of the state is then:

$$\bar{n} = \sum_n n \mathcal{P}(n) = 0 \cdot \mathcal{P}(0) + 1 \cdot \mathcal{P}(1) = \frac{e^{-\frac{\epsilon - \mu}{kT}}}{1 + e^{-\frac{\epsilon - \mu}{kT}}} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}$$

It is the **Fermi-Dirac distribution**:  $\bar{n}_{FD} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}$ .



# Bose-Einstein distribution

If the particles are bosons, then  $n$  can be any nonnegative integer, so the grand partition function is:

$$\mathcal{Z} = 1 + e^{-\frac{\epsilon - \mu}{kT}} + e^{-\frac{2(\epsilon - \mu)}{kT}} + \dots = 1 + e^{-\frac{\epsilon - \mu}{kT}} + \left(e^{-\frac{\epsilon - \mu}{kT}}\right)^2 + \dots = \frac{1}{1 - e^{-\frac{\epsilon - \mu}{kT}}}$$

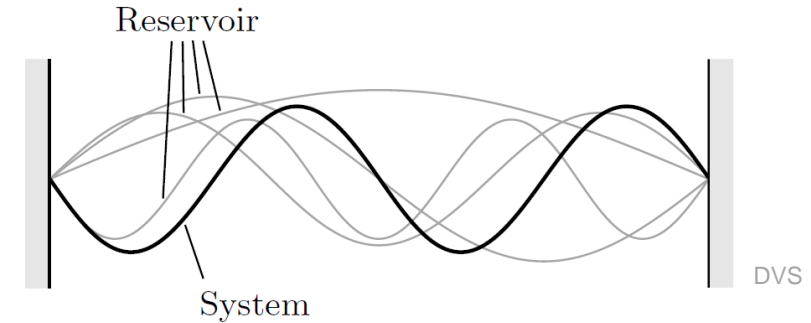
$$\mathcal{P}(n) = \frac{1}{\mathcal{Z}} e^{-\frac{n\epsilon - \mu n}{kT}} = \frac{1}{\mathcal{Z}} e^{-\frac{n(\epsilon - \mu)}{kT}}$$

The average number of particles in the state or the occupancy of the state is then:

$$\bar{n} = \sum_n n \mathcal{P}(n) = 0 \cdot \mathcal{P}(0) + 1 \cdot \mathcal{P}(1) + 2 \cdot \mathcal{P}(2) + \dots = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} - 1}$$

This is **Bose-Einstein distribution**:  $\bar{n}_{BE} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} - 1}$

Like the Fermi-Dirac distribution, the Bose-Einstein distribution goes to zero when  $\epsilon \gg \mu$ .  
Unlike the Fermi-Dirac distribution, it goes to infinity as  $\epsilon$  approaches  $\mu$  from above.





# Comparison of distributions

For the Boltzmann distribution:  $P(s) = \frac{1}{Z_1} e^{-\frac{\epsilon}{kT}}$       $\mu = -kT \ln \left( \frac{Z_1}{N} \right)$

$$\bar{n}_{Boltzmann} = \frac{1}{Z_1} N e^{-\frac{\epsilon}{kT}} = e^{-\frac{\epsilon}{kT}} e^{\frac{\mu}{kT}} = e^{-\frac{(\epsilon-\mu)}{kT}}$$

$$F = -kT \ln(Z)$$

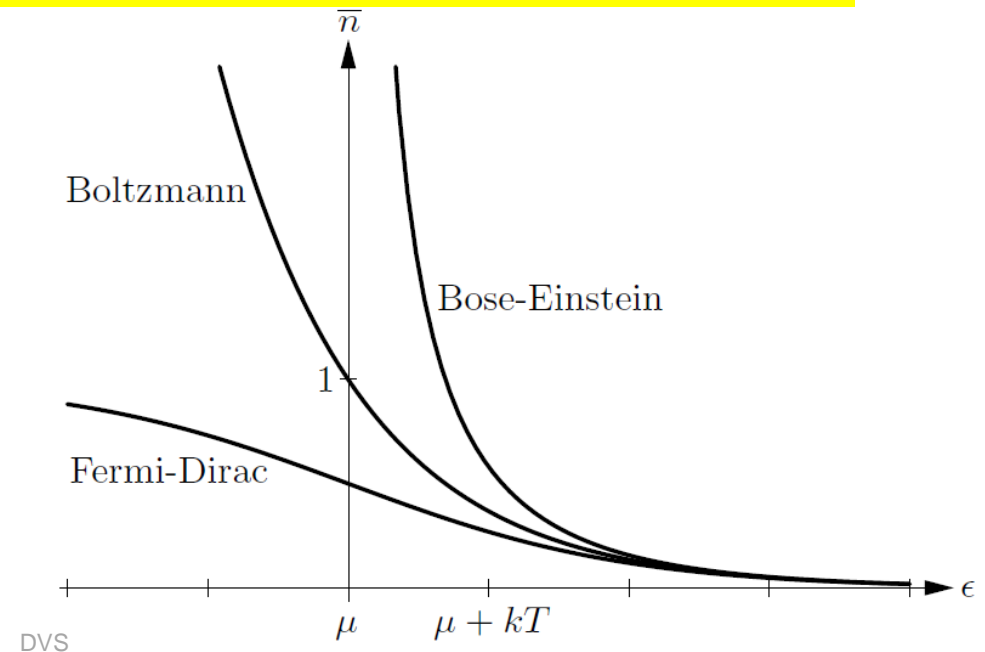
$$\mu = + \left( \frac{\partial F}{\partial N} \right)_{T,V}$$

$$Z = \frac{Z_1^N}{N!} = \frac{Z_1^N}{N!}$$

$$\ln N! \approx N(\ln N - 1)$$

$$\bar{n}_{Boltzmann} = e^{-\frac{(\epsilon-\mu)}{kT}} \quad \bar{n}_{FD} = \frac{1}{e^{\frac{\epsilon-\mu}{kT}} + 1} \quad \bar{n}_{BE} = \frac{1}{e^{\frac{\epsilon-\mu}{kT}} - 1}$$

When  $\epsilon \gg \mu$ , the exponent is very large, one can neglect the 1 in the denominator of Fermi-Dirac and Bose-Einstein distributions, and both are reduced to the Boltzmann distribution. The precise condition for the three distributions to agree is:  $\epsilon - \mu \gg kT$ .

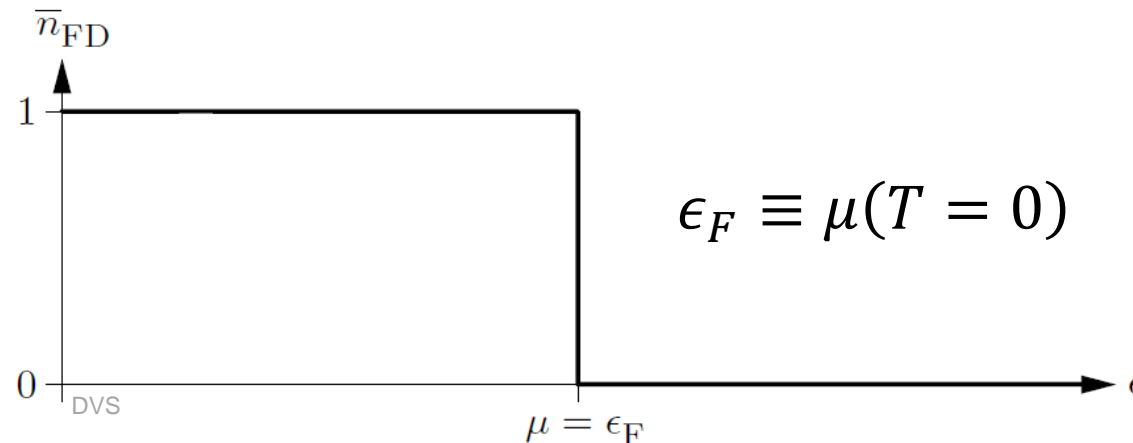


# Degenerate Fermi gas

- Gas of fermions is degenerate when **nearly all states below  $\mu$  are occupied and nearly all states above  $\mu$  are unoccupied**, which typically happens at a low temperatures  $kT < \epsilon - \mu$ .
- At zero temperature, Fermi-Dirac distribution function is a step function. It equals **1 for all states with  $\epsilon < \mu$  and equals 0 for all states with  $\epsilon > \mu$** .
- As a boundary of filled state at  $T = 0$ ,  $\mu$  is also called Fermi energy:  $\epsilon_F$ .
- The value of  $\epsilon_F$  is determined by the total number of electrons.
- **All electron states are filled** from the lowest available state to  $\epsilon_F$ .
- **$\epsilon_F$  is change in total energy** at zero temperature when **one particle is added** to the system.

$$\bar{n}_{FD} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}$$

$$\mu = \left( \frac{\partial U}{\partial N} \right)_{S,V}$$



Counting quantized states in 3D:

$$\epsilon_F = \frac{h^2}{8m} \left( \frac{3N}{\pi V} \right)^{\frac{2}{3}}$$

# Properties of degenerate Fermi gas

- The average energy of the electrons is  $3/5$  the Fermi energy:  $U = 3/5\epsilon_F$ . Fermi energy for conduction electrons in a typical metal is a few electron-volts. This is much larger than the average thermal energy of a particle at room temperature,  $kT \approx 1/40 eV$ , which means electron gas in metals is a degenerate Fermi gas.
- The condition  $\epsilon_F \gg kT$  comes from the condition  $V/\vartheta_Q \ll N$ , which means that quantum statistics is important for the electron gas.
- The large, comparable with  $kT$ , Fermi energy justifies approximation of  $T \approx 0$ .
- Using the formula  $P = -(\partial U/\partial V)_{S,N}$ , the degeneracy pressure  $P = \frac{2U}{3V}$  is found to be few billion  $N/m^2$ , sufficient to withstand electrostatic forces. This pressure does not come from the electrostatic repulsion between the electrons. It arises purely from the quantum exclusion principle.
- All electron states are filled from the lowest available state to  $\epsilon_F$ .

# Fermi gas at small nonzero temperatures

- At **finite temperature  $T$** , normal particles would get energy about  $kT$ . However, degenerate electron gas is special. Most of the electrons cannot acquire such energy, because all the states that they might jump in are already occupied.
- The only electrons that can acquire some energy (thermal) are those that are already within  $kT$  of the Fermi energy. Only they can jump up into unoccupied states above  $\epsilon_F$ .
- The number of electrons that can be affected by the increase in  $T$  is proportional to  $T$ . This number must also be proportional to  $N$ . Thus, the **additional energy at finite  $T$**  is doubly proportional to  $T$ :  $\Delta U(T) \propto NkT \cdot kT$ .
- Coefficient proportionality can be guessed from dimensionality units. It must have unit of one over energy, and the only energy available in this model is  $\epsilon_F$ .
- Knowing this, allows to calculate **heat capacity** of electron gas. It is going to zero as  $T \rightarrow 0$ .

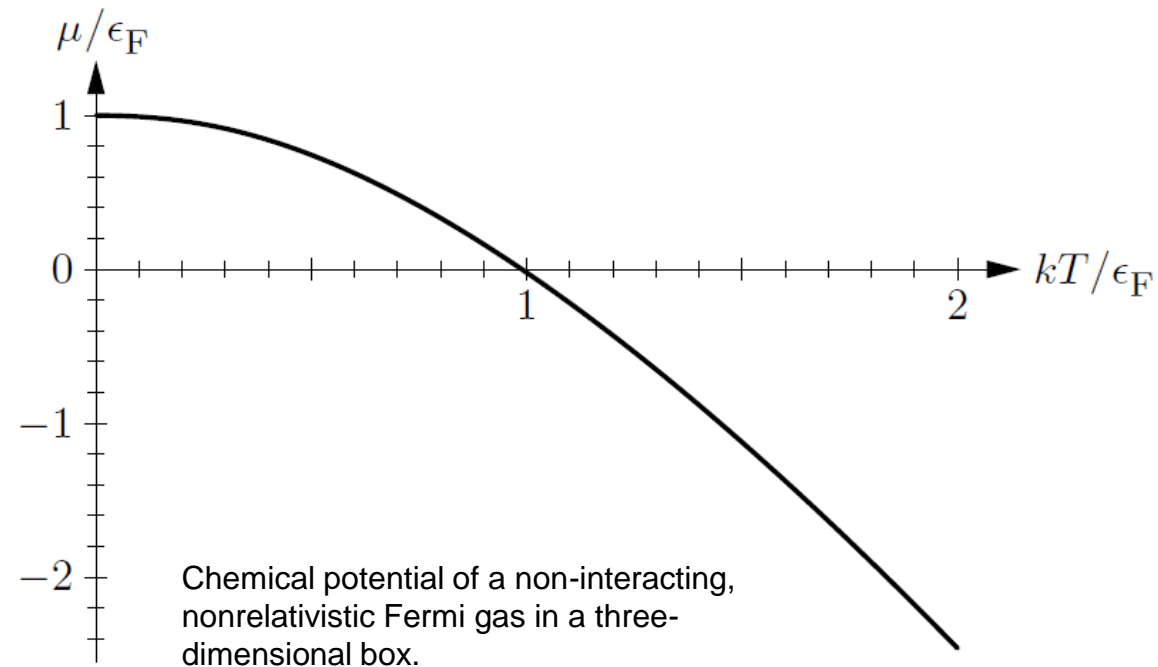
$$U = \frac{3}{5} N \epsilon_F + A \frac{NkT \cdot kT}{\epsilon_F} \quad A = \frac{\pi^2}{4} \quad C_V = \left( \frac{\partial U}{\partial T} \right)_V = \frac{\pi^2 Nk^2 T}{2\epsilon_F}$$

# Chemical potential of degenerate Fermi gas

- The chemical potential,  $\mu$ , is the point where the **probability of a state being occupied is exactly 1/2**.
- At  $T = 0$ ,  $\mu = \epsilon_F$ .
- The chemical potential **decreases with increase of  $T$** .
- At high temperatures,  $\mu$  becomes **negative** and approaches the form for an ordinary gas obeying Boltzmann statistics.

Chemical potential of ideal gas:

$$\mu = -kT \ln \left[ \frac{V}{N} \left( \frac{2\pi m kT}{h^2} \right)^{3/2} \right]$$



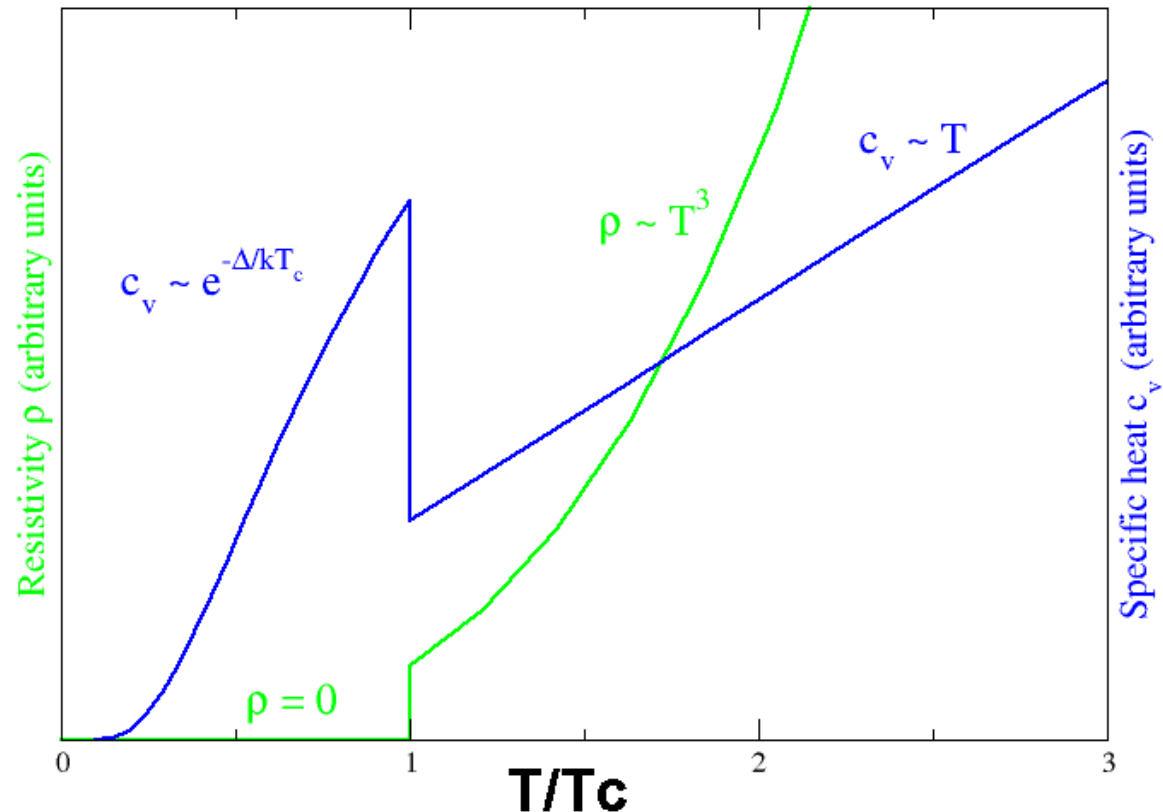
Chemical potential of a non-interacting, nonrelativistic Fermi gas in a three-dimensional box.

# Bose-Einstein condensation and superconductivity

- Superconductivity is the result of **Bose-Einstein condensation** taking place when fermions form bosons being united into Cooper pairs.
- As a result, electron gas acquires property of **superfluidity** dropping **resistance to absolute zero**.
- Superconductors have **unique quantum properties** allowing multiple uses in **modern technology**.

Fermi gas: 
$$C_V = \left( \frac{\partial U}{\partial T} \right)_V = \frac{\pi^2 N k^2 T}{2 \epsilon_F}$$

Superconductor:



# Summary

- **Chemical potential** is a key for understanding behavior of **electron gas**.
- Chemical potential **changes from negative to positive** at a transition from **classical to quantum** behavior.
- **Electrons in a metal behave quantum mechanically** with positive chemical potential.
- **Quantum behavior** results in quantum statistics: **Bose-Einstein and Fermi-Dirac**.
- Electrons in a metal **at room temperature** are well described by a model of **degenerate Fermi-Dirac gas**. In this gas, **all states are filled at energies below the Fermi energy  $\epsilon_F$  and empty above**.
- Chemical potential of degenerate Fermi-Dirac gas **changes from positive to negative when  $kT$  becomes higher than  $\epsilon_F$** .
- **Electrons** can be **Bose-Einstein particles** when they are united into Cooper pairs, which leads to phenomenon of **superconductivity**.