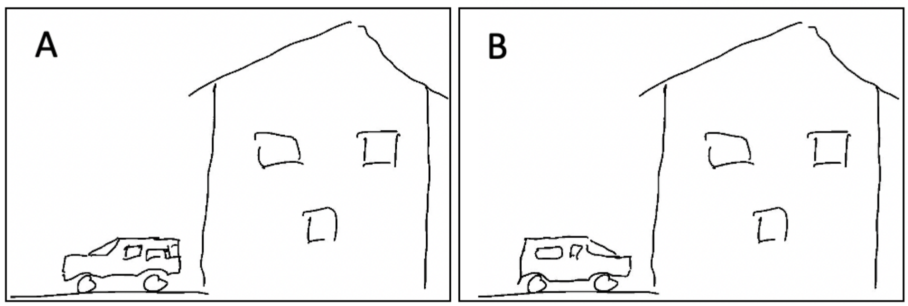
Exercise week 34: FYS2160, Thermodynamics and statistical physics, Fall 2022

1 Icy windshield



I park my car in front of my house and leave it overnight when there is a clear sky in winter. When the car has the front facing away from the house the windshield is icy in the morning, but not when the front is facing the house. Why does the windshield become icy? Why does it matter which way the car faces?

Make a sketch of the different processes at work this winter night. Describe in words the different processes you consider to answer the questions.

Solution:

- The windshield gets icy when it is cold ($T < 0$) and there is water vapour in the air at a higher chemical potential (vapour pressure) than on the windshield.
- The chemical potential in the air is regulated by the temperature of other surfaces around and by transport of (moist air) from the atmosphere
- The air circulates and the vapour pressure of the air is affected by the temperature of other surfaces in the vicinity. The distance to other surfaces like the house will (due to circulation) not affect the vapour pressure of the air by the windshield very much.
- Thus the difference between the two cases (facing towards or away from) must be due to the difference in the temperature of the windshield
- The temperature is affected by radiation, conduction and convection. Since the air has the same temperature and humidity in the two cases he conductive and convective heat loss should not be much different in the two cases, thus it is mainly an effect of radiation balance.

- There are two bodies radiating towards the car: the clear sky and the house.
- The clear sky has a background radiation of 4K and radiation from CO2 and H2O in the atmosphere (on average 255K according to Schroeder p 308, but much lower on a winter night).
- There outer surface of the house is probably slightly warmer than the air surrounding it (due to heat conduction through the walls).
- The cold, icy windshield of the car facing away from the house radiates less than the warmer, windshield facing the house, but still the difference in received radiation is more than large enough to offset this.

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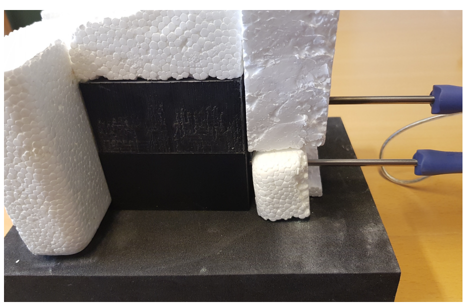
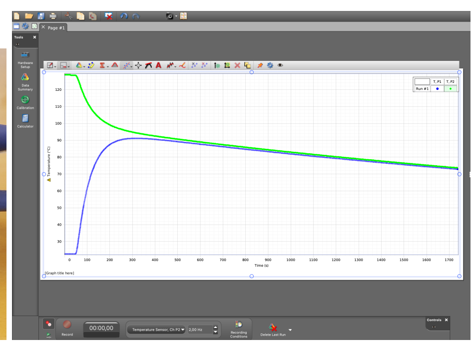
2 Thermal Concepts Inventory quiz

On the the lecture 1 webpage you will find your answers to the quiz. For many of the questions there was serious disagreement about what the correct answer was. For each of these, discuss whether there was one correct answer or if several answers were acceptable.

3 Heat conduction experiment

In the second lecture a heat conduction experiment was performed, placing a hot metal block on top of a cold metal block. Both metal blocks have dimensions 38 mm x 38 mm x 75 mm. One metal block was made from S235JR steel and the other from lead. The temperature was measured by thermometers inside the metal blocks starting before the blocks were put into thermal contact. The thermometers were in the middle of the metal blocks.

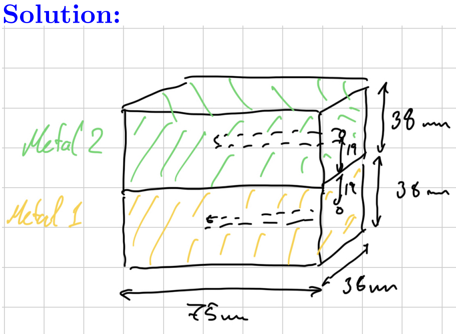
Make a sketch of the experiment with the essential information needed to model it and explain which simplifying assumptions you make.

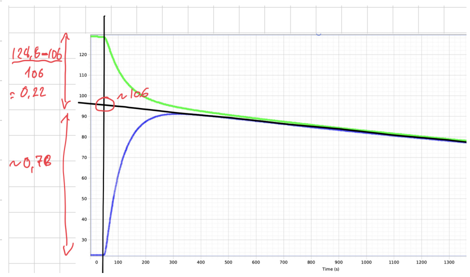
Solve the thermal diffusion equation

$$\frac{\partial T}{\partial t} - \frac{\lambda}{c\rho}\nabla^2 T = 0$$

with the appropriate boundary and initial conditions and compare your result with the data from the experiment In the thermal diffusion equation T is temperature, t is time, λ is the thermal conductivity, c is the specific heat capacity, ρ is the density and z is the position coordinate. You may choose to solve the diffusion equation analytically or numerically. Try to explain the sources of deviation of the model data from the experiment.

Solution:





A sketch of the experimental situation is given above. The temperature difference initially was only vertical. There was some isolation on the sides to avoid heat loss to environment. If we assume that the heat loss was minimal the whole thermal conduction problem is one dimensional. We will also assume that there is no extra resistance to thermal conduction at the interface between the two materials. That means that there the conduction is governed solely by the factors $\lambda/(c\rho)$ of the two materials.

I have drawn a line following the heat loss from the two blocks back to the moment when they were put in contact. Thus without heat loss to the environment the final temperature of the experiment would be around 95-100C (error in the notes on the drawing). Thus the lower block has a volumetric heat capacity which is much smaller than the upper block.

The properties of lead and steel are

	ρ [g/cc]	c [J/(g K)]	$c\rho$ [J/(cc K)]	λ [W/(m K)]	$\frac{\lambda}{c\rho}$ [m^2/s]
Lead	11.34	0.13	1.47	35	$2.4 \cdot 10^{-5}$
Steel	7.8	0.47	3.66	42.5	$1.2 \cdot 10^{-5}$

We can conclude that the lead block is at the bottom and the steel block is on top.

The numerics can be simplified if one rescales the variables by their characteristic values:

$$T' = 2\frac{T - \bar{T}_0}{\Delta T_0}, \bar{T}_0 = 75.7C, \Delta T_0 = 106.2C$$

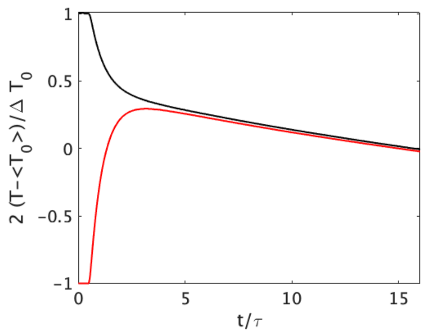
$$z' = z/l, l = 0.038m$$

$$t' = t/\tau, \tau = 100s$$

The one dimensional thermal conduction can then be summarised by the following:

$$\frac{\partial T'}{\partial t'} - \alpha \frac{\partial^2 T'}{\partial z'^2} = 0,$$

with $\alpha = \frac{\lambda \tau}{c \rho l^2} = 0.83$ for $z > 0$ and $\alpha = 1.66$ for $z < 0$. The initial condition is $T = 0.5$, $z > 0$ and $T = -0.5$, $z < 0$. The boundary conditions are that there is no heat flow at the top and bottom boundaries. Since $J_Q \propto \frac{\partial T}{\partial z'}$ the boundary conditions can be expressed: $\frac{\partial T}{\partial z'} = 0$ at $z' = 1$ and $z' = -1$.



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4 Work and pressure

Problem 1.34 in Schroeder

Solution:

Problem 1.34. It's easiest to first compute the work done during each step, using $W = -P\,\Delta V$. For steps A and C there is no work done because the volume doesn't change; for steps B and D the pressure is constant so we don't need to set up an integral. So, for instance, the work done on the gas during step D is $+P_1(V_2 - V_1)$.

Since each molecule has five degrees of freedom, the thermal energy of the gas at any point is $U = \frac{5}{2}NkT = \frac{5}{2}PV$. Therefore ΔU during any step is $\frac{5}{2}(P_f V_f - P_i V_i)$, where f stands for final and i stands for initial. For instance, during step D, $\Delta U = -\frac{5}{2}P_1(V_2 - V_1)$.

The heat added to the gas during any step is just $Q = \Delta U - W$. So again for step D we have $Q = -\frac{5}{2}P_1(V_2 - V_1) - P_1(V_2 - V_1) = -\frac{7}{2}P_1(V_2 - V_1)$.

Here's a table of values for all four steps, computed in this way:

	W	ΔU	Q
step A:	0	$\frac{5}{2}V_1(P_2 - P_1)$	$\frac{5}{2}V_1(P_2 - P_1)$
step B:	$-P_2(V_2 - V_1)$	$\frac{5}{2}P_2(V_2 - V_1)$	$\frac{7}{2}P_2(V_2 - V_1)$
step C:	0	$-\frac{5}{2}V_2(P_2 - P_1)$	$-\frac{5}{2}V_2(P_2 - P_1)$
step D:	$P_1(V_2 - V_1)$	$-\frac{5}{2}P_1(V_2 - V_1)$	$-\frac{7}{2}P_1(V_2 - V_1)$
whole cycle:	$-(P_2 - P_1)(V_2 - V_1)$	0	$(P_2 - P_1)(V_2 - V_1)$

I found the entries in the last row by adding up each of the columns and simplifying the result as much as possible.

What's actually happening must be something like the following: During step A we hold the piston fixed but put heat in (say from a flame); during step B we let the piston out and continue putting heat in at such a rate as to maintain constant pressure; during step C we hold the piston fixed but suck heat out, perhaps by immersing the whole thing in an ice bath; and during step D we push in the piston while still sucking heat out so the pressure again remains steady.

The net work done on the gas during the whole cycle is negative; in other words, the net work done *by* the gas is positive. This is as expected, because the pressure is higher when the gas is expanding than when it is being compressed. Notice that the net work is just minus the area enclosed by the rectangular cycle on the diagram. The net change in the energy of the gas is zero, as it must be: the state of the gas (as determined by its pressure and volume) is the same at the end of a cycle as at the beginning. Therefore the net heat put into the gas must be minus the net work done, as indeed it is. In summary, this procedure results in a net conversion of heat input into work output.