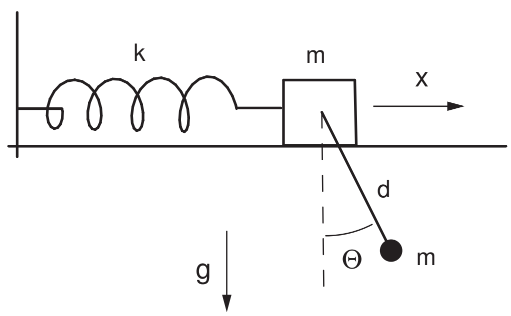
FYS 3120/4120: Classical Mechanics and Electrodynamics

Spring semester 2006

Homework (voluntary), March 2-9.

Written solutions that are returned before the Thursday exercises, March 9, will be corrected and commented.
Solutions can be returned to the office Ekspedisjonskontoret in the Physics building.
For questions concerning the problems, please contact Mats Horsdal or Jon Magne Leinaas.
The problem set consist of 3 problems printed on 3 pages.
Solutions may be written in either Norwegian or English.

PROBLEM 1
A composite mechanical system.



A composite system is shown in Figure 1. It consist of a block of wood with mass m connected to a spring with spring stiffness k and a pendulum attached to the block. The block moves without friction on a horizontal table and the pendulum oscillates freely under the influence of gravity. The pendulum sphere has the same mass m as the block and the length of the pendulum rod is d. The spring as well as the pendulum rod is considered as massless. The horizontal displacement x of the block and the angular coordinate θ of the pendulum are chosen with $x = 0$ and $\theta = 0$ at equilibrium. All motion is considered to be in a plane (the x,y-plane) as indicated in the figure.

a) Find the expression for the Lagrangian of the system.

b) Derive the equations of motion of the system with x og θ as generalized coordinates.

c) Find expressions for the generalized momenta p_x and p_θ corresponding to the variables x and θ. Find the velocities $\dot{x}$ and $\dot{\theta}$ expressed in terms of the generalized coordinates and momenta.

d) Find the Hamiltonian of the system and the corresponding Hamilton's equations.

PROBLEM 2
Relativistic transformations.

A light signal is sent from a space-time point A with coordinates $x = y = z = t = 0$ in an inertial frame S. It is received at another point B with coordinates $x = d\sin\theta, y = d\cos\theta, z = 0, t = d/c$, where d is the distance between the two points in S, $\theta = 30^o$ is the direction of the light signal relative to the x-axis, and c is the speed of light. The wave length of the light is $\lambda = 400$ nm measured in S.

Another inertial frame S' is moving with velocity $v = 3/5\,c$ in the x-direction relative to S. The space axes of the two frames are chosen to be parallel and the origin of S' is chosen so that the point of emission A also in this reference frame has coordinates $x' = y' = z' = t' = 0$.

a) At what angle θ' is the light signal sent relative to the x'-axis of reference frame S'.

b) What is the wave length λ' of the light signal in reference frame S'? (Hint: It may be useful to think of the distance between A and B as corresponding to an integer number of wave lengths, $d = n\lambda$. Why is a similar expression valid in S'?)

A muon is an unstable particle with a life time which we denote by τ_0. We consider a situation where two muons are created at the same time and the same point in space, specified by the coordinates $(x, y, z, t) = (0, 0, 0, 0)$ in an inertial frame S. We assume the first muon is created at rest and disintegrates after the time τ_0. The other muon is created with velocity $v = 3/5\,c$ in the x-direction. Also this particle disintegrates after a time τ_0, but measured in the co-moving rest frame S' rather in the reference frame S.

c) What are the coordinates x and t of the point of disintegration for the moving muon, as measured in S? (Express x and t in terms of τ_0 and c.)

d) Draw a Minkowski diagram (x,ct)-diagram for the reference frame S, which shows the space-time paths with end points (creation and disintegration) for the two muons. Make the drawing accurate with respect to the relative life times of the two particles and the directions of the space-time paths.

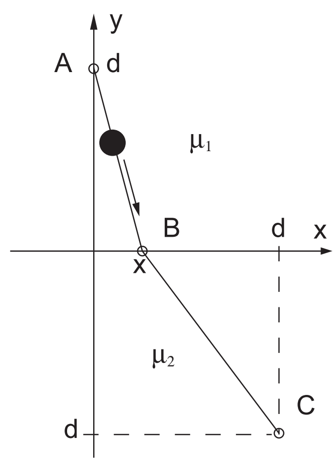
PROBLEM 3
A variational problem.

A body is placed on a horizontal table, as shown in Fig. 2. The table is coordinized by a set of Cartesian coordinates (x, y). The friction coefficient (kinetic friction) depends on the position on the table, $\mu = \mu(x, y)$.

Consider the following problem. The body is moved from a point A with coordinates (x_0, y_0) to a point B with coordinates (x_1, y_1) in such a way that the energy loss along the path, due to friction, is minimized.

a) Formulate the problem as a variational problem.

Make next the assumption that the friction coefficient μ has a constant value μ_1 in the upper half plane, $y > 0$, og and another constant value μ_2 in the lower halfplane, $y < 0$. Make further the assumption $\mu_2 = \mu_1/2$. The coordinates of A are now specified as $(0, d)$ and the coordinates



of B as $(d, -d)$ with d as an unspecified length.

b) Explain why the path from A to B is composed of two straight lines, one in the upper and one in the lower half plane. Show how to reduce the variational problem to the problem of finding the point (with coordinate x) where the two lines are connected.

c) Show that finding this x-coordinate can be formulated as solving a forth order equation in x. Solve this problem graphically by plotting the fourth order polynomial as a function of x. Express the solution in terms of the length d.