# Problem Set 12

## Problem 12.1

In this problem we will study the radiation field from an antenna of length $a$, which is lying along the $z$-axis with endpoints at $z = \pm a/2$. The antenna is carrying a current given by

$$I = I_0 \cos(\pi z/a)\cos(\omega t).$$

a) Let $\alpha(z,t)$ be the charge per length. Derive the expression

$$\frac{\partial \alpha}{\partial t} = -\frac{\partial I}{\partial z}$$

and show that (ignore any static term)

$$\alpha = \frac{\pi I_0}{a\omega}\sin\left(\frac{\pi z}{a}\right)\sin(\omega t).$$

b) Show that the electric dipole moment is given by

$$\mathbf{p} = \frac{2aI_0}{\pi\omega}\sin(\omega t)\mathbf{e}_z$$

and that the magnetic moment about the origin is zero.

c) In our case $\mathbf{p}$ is independent of the reference point. Which property of the charge distribution is this related to? Show that $d\mathbf{p}/dt$ and $d^2\mathbf{p}/d^2t$ for any charge distribution is independent of the reference point.

d) Assume that the wavelength of the emitted radiation is much greater than the length of the antenna and express this as a relation between the given constants. Most of the radiation in the wave zone will then be electrical dipole radiation. Write down the expressions for $E$ and $B$ in the radiation zone in this approximation. Make a sketch of the fields. Is the wave linearly or circularly polarized?

## Problem 12.2

Figure 1 shows a straight antenna of length $2a$ lying along the $z$-axis with its center in the origin. We assume that the charge of the antenna is located at the endpoints. The current in the 'bulk' of the antenna is then given by $I = I_0 \sin\omega t$ where $\omega$ and $I_0$ are constants. The antenna is electrical neutral at time $t = 0$. The field point is given by the position vector $\mathbf{r}$ and in spherical coordinates $(r, \theta, \phi)$.

a) Show that the antenna's electrical dipole moment at time $t$ is given by $\mathbf{p}(t) = \frac{2aI_0}{\omega}(1 - \cos\omega t)\mathbf{k}$, where $\mathbf{k}$ is the unit vector in the $z$-direction.

We will now assume that the fields can be treated as electrical dipole radiation.

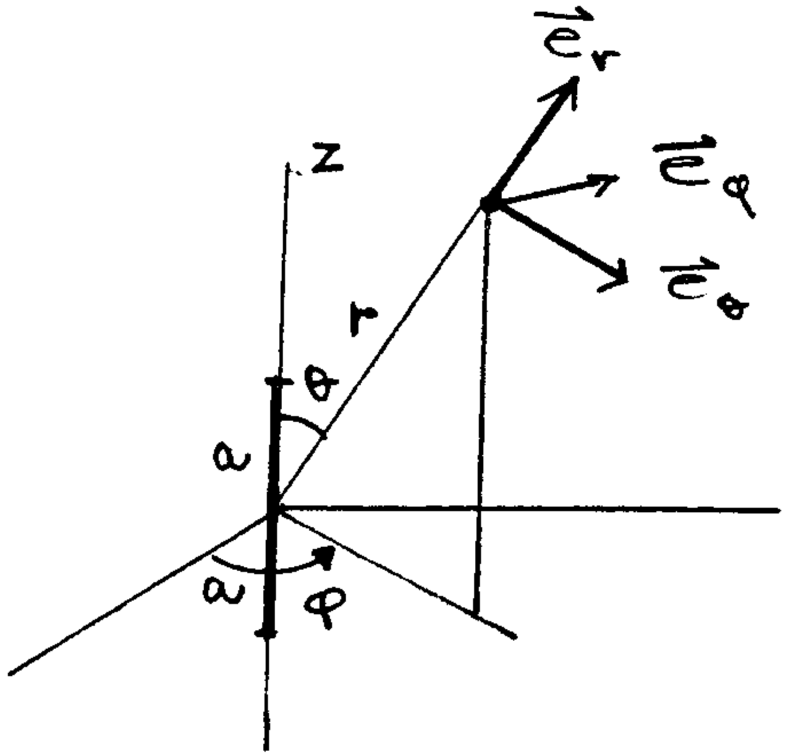


Figure 1: The geometry in Problem 12.2

b) Find the components of the **B** and **E** fields in the directions $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ in the field point $(r, \theta, \phi)$ at time $t$.

c) Show that the time average of the total radiated power in all directions kan be written as $\langle P \rangle = \frac{RI_0^2}{2}$ and find $R$ (radiation resistance). What is the time average of the total power consumed by the antenna if it has an 'ordinary' resistance $R_0$ as well?

d) Find $R$ for an antenna of length $2a = 5$ cm which is conducting a current with frequency $f = 150$ MHz. What is the time average of the total radiated power when $I_0 = 30$ A?