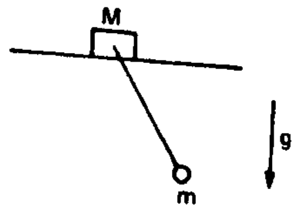

Figure 1: Problem 3.1a)

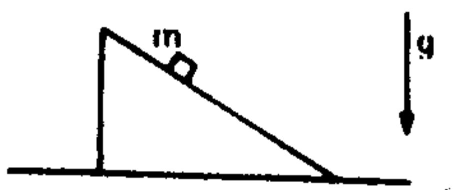

Figure 2: Problem 3.1b)

Problem Set 3

Problem 3.1

Choose suitable generalized coordinates for the systems, and find Lagrange's equations of motion. Try to interpret the equations in terms of other mechanical principles. Search for exact solutions and constants of motion.

a) A mathematical pendulum connected to a box that can slide without friction on a horisontal plane. (Assume that the motion takes place in the vertical plane.) (Figure 1).

b) A particle that slides without friction on a tilted plane. The body that constitutes the the tilted plane is forced to to move horisontally with an acceleration a. (Figure 2)

c) A rod in the horisontal plane is forced to move in such a way that the end points are in contact with the coordinate axes. The angle φ increases linearly with time. A particle slides without friction along the rod. (Figure 3).

d) A rigid, smooth rope shaped as a circle rotates with constant angular velocity along a diameter. A particle slides without friction along the circle and there is no gravity (Figure 4).

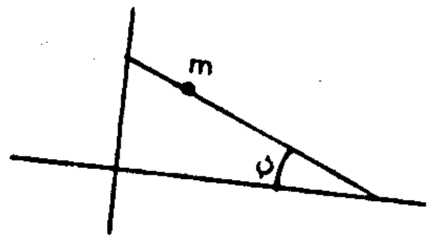

Figure 3: Problem 3.1c)

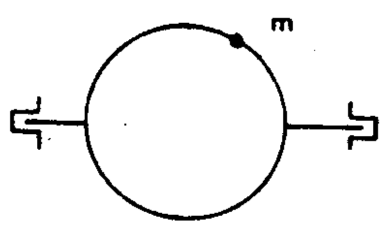

Figure 4: Problem 3.1d)

Problem 3.2

To bodies with the same mass, m, are connected with a massless rope through a small hole in a smooth horisontal plane. One body is moving on the plane, the other one is hanging at the end of the rope and can move verically. At all instances the rope is taut (i.e. not slack). The acceleration due to gravity is g.

a) Find the Lagrange's equations of motion in polar coordinates (r, θ) and explain their physical meaning.

b) Reduce the equations of motion to a one dimensional problem in r with an effective potential and discuss the motion. Discuss special cases.