Problem Set 4

Problem 4.1

Figure 1 shows a rod of length b and mass m. One endpoint of the rod is constrained to move along a horisontal line and the other endpoint along a vertical line. The two lines are in the same plane. There is no friction and the acceleration due to gravity is g.

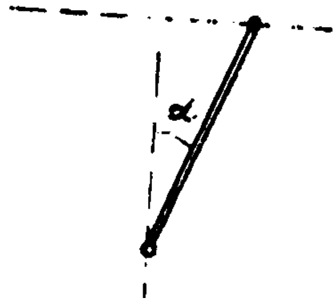

Figure 1: Problem 4.1

a) Find Lagrange's equations with α as coordinate.

b) Find the period for small oscillations about the equilibrium position.

c) Find the period for oscillations with amplitude $\pi/2$.

Problem 4.2

A particle of mass m is attached to the circumference of a circular disc of radius r. The disc can perform a rolling motion on the underside of a horisontal line, see Figure 2. We assume that the disc is massless and that the motion takes place in a vertical plane. Find the Lagrangian, first with ϕ, then with s as a generalized coordinate. Find the period of motion.

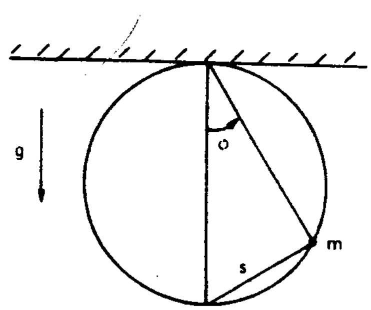

Figure 2: Problem 4.2

Problem 4.3

A particle of mass m and charge q is moving in a magnetic field given by the vectorpotential (in polar coordinates)

$$A_r = A_\theta = 0 \qquad\qquad A_\phi = \frac{k}{r}\tan(\theta/2),$$

where k is a constant. Throughout this problem we will use polar coordinates $(r,\, \theta,\, \phi)$ and assume that the motion is non-relativistic.

a) Find the **B**-field. Try to find a way in which it can be (approximately) realised.

b) Find the Lagrangian and Lagrange's equations.

c) Show that the kinetic energy is a constant of motion.

d) Explain the physical meaning of Lagrange's equation for r.

e) Show that there exists solutions of the form

$$r = (a^2t^2 + b^2)^{1/2} \qquad\qquad \theta = \theta_0,$$

where a, b,and θ_0 are constants.

f) Give a physical enterpretation of the constants a and b.

g) Make a sketch that shows the magnetic field and a trajectory of the type we have just found.