Problem Set 5

Problem 5.1 (Exam 1992, exercise 2)

Figure 1 shows a circular cone with vertical axis z and opening angle 2α. A particle of mass m slides without friction on the inner surface of the cone. The particle's position is given by the coordinates (r, θ). The acceleration due to gravity is g.

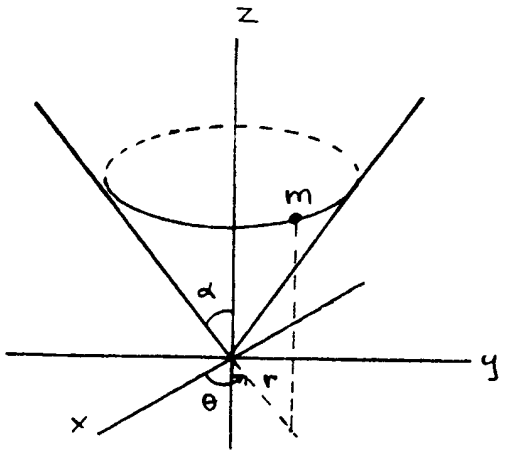

Figure 1: Problem 5.1

a) Show that the Lagrangian for the particle is

$$L = \frac{1}{2}m(\dot{r}^2(1 + \cot^2\alpha) + (r\dot{\theta})^2) - mgr\cot\alpha.$$

b) Find Lagrange's equations for the particle.

c) Find two constants of motion and explain their physical meaning.

d) Which initial velocity $\mathbf{v}_0$ must be given to the particle in the point $(r = r_0, \theta = 0)$ to make it move in a horisontal trajectory? Show that the answer can be found from the equations of motion as well as from elementary ideas from Newtonian mechanics?

e) Show that the Hamiltonian is given by

$$H = \frac{p_r^2}{2m(1 + \cot^2\alpha)} + \frac{p_\theta^2}{2mr^2} + mgr\cot\alpha.$$

f) Show how the equations of motion that was found in part b) also can be found from Hamilton's equations.

g) Show how the two constants of motion in part c) can be found from the Hamiltonian formalism.