Problem Set 6

Problem 6.1

This problem is an exercise in using the postulates of special relativity in an elementary way. The use of Lorentz transformations should therefore be avoided.

Assume that we have a train carriage moving in a straight line with constant velocity v. The earth is considered as an inertial system (S). The length of the carriage in S (when the velocity is v) is L. Figure 1 shows the carriage when it is simultaneously measured in S. A and B are points on the rear wall and front wall, respectively. C is a point in the middle of the carriage.

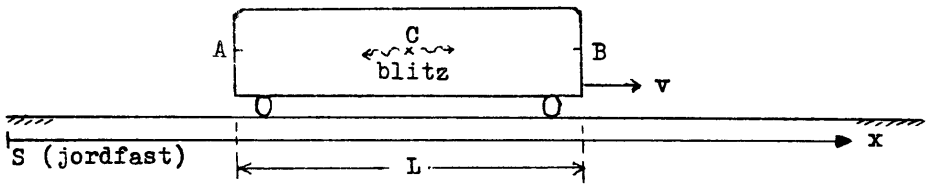

Figure 1: The train carriage in problem 6.1.

a) In Figure 2 we have drawn the world line for point C. Draw the worldlines for the points A and B. Show that the angle α between these lines and the time axis is given by $\tan\alpha = v/c$.

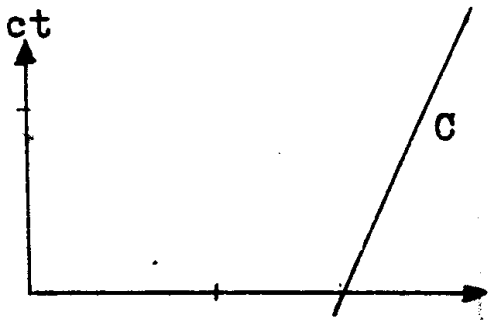

Figure 2: The Minkowski diagram in Problem 6.1c).

At a given time a flash tube is discharged in point C. We will call this event E_0. Some of the light will propagate backward in the carriage and some will propagate forward. Let E_1 and E_2 be the events that the light hits the rear wall and front wall, respectively. Let us assume that the light is reflected from A and B, and let E_3 be the event that the reflected light pulses meet. (We assume that the light is moving along the length of the carriage.)

b) Draw the world lines of the light signals as well as the four events E_0, E_1, E_2 and E_3 in the Minkowski diagram.

c) Explain why E_1 and E_2 are simultaneous in the reference frame of the carriage and that E_0 and E_3 are at the same place in this reference system.

d) Draw the straight line from from E_1 to E_2 and show that the angle between the x-axis and this line is α. (This is geometrical problem).

e) Show that a signal that connects E_1 and E_2 must have the velocity c^2/v (which is greater than c).

f) Let E be any event, i.e. any point in the Minkowski diagram. Plot the events in the carriage system that are simultaneous with E. Plot the events in the carriage system that occur at the same place as E.

In the following we let S' denote the reference system of the carriage. Let x' and t' be chosen such that $x' = 0$, $t' = 0$ corresponds to $x = 0$, $t = 0$ (i.e. a homogeneous transformation).

g) Plot the points in the Minkowski diagram where $t' = 0$ (the origin is one of them) and $x' = 0$.

h) The set of events plotted in g) constitutes a skewed coordinate system for x' and ct'. The space-time position for any event can be read out in the orthogonal coordinate system as x and ct and in the skewed system as x' and ct'. Explain how.