Problem Set 8

Problem 8.1

a) Give a short definition of "inertial system" and "the instantaneous rest frame of a particle".

b) Use the expression for the four acceleration of a particle with velocity **u** and acceleration **a** to show that for a particle moving in a circle with constant velocity u we have

$$A_\mu A^\mu = \gamma^4 a^2.$$

Also show that

$$a = \gamma^{-2} a^0,$$

where a^0 is the particles acceleration in its instantaneous rest frame.

Figure 1 shows a particle at rest with rest mass M_0 in an inertial system S^0. The particle emits a photon and the rest mass is reduced to m_0. The particle's momentum and energy after the emission is p and E, respectively, and the photon's momentum and energy is p_1 and E_1, respectively.

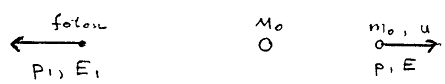

Figure 1: The emission process described in problem 8.1.

c) Write down E, p_1 and E_1 expressed in terms of p and m_0. Find p and E expressed in terms of M_0 and m_0 and show that the particle's velocity u is given by

$$\frac{u}{c} = \frac{1 - (m_0/M_0)^2}{1 + (m_0/M_0)^2}$$

d) Show that for $|\Delta M_0| \ll M_0$, where $\Delta M_0 \equiv m_0 - M_0$, we have that

$$u = -c\frac{\Delta M_0}{M_0}.$$

Figure 2 shows a "photon rocket" which is being driven by the recoil of a continous stream of photons. The rocket's restmass at any instance is m_0.

e) Use the previous result to show that the rocket's acceleration a^0 in it's instantaneous rest frame is given by

$$a^0 = -\frac{c}{m_0}\frac{dm_0}{d\tau},$$

where dm_0 is the change of the rocket's rest mass during the proper time interval $d\tau$.

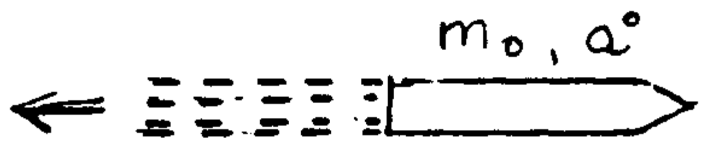

Figure 2: The "photon rocket" in problem 8.1.

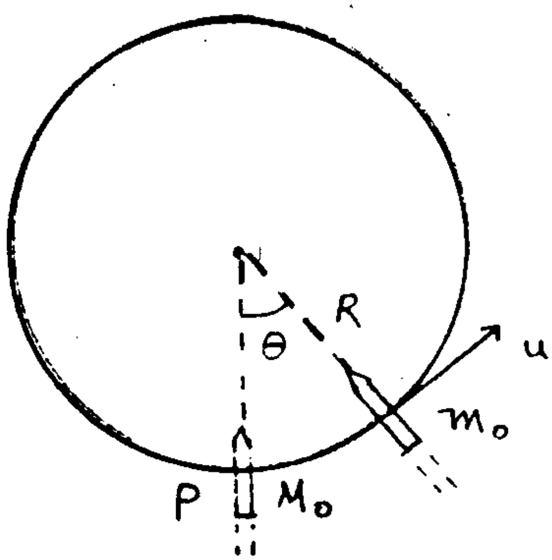

Figure 3: The circular motion of the rocket.

By the use of the rocket engine the rocket is maneuvered so that it will perform circular movement with velocity u in an inertial frame S. See Figure 3.

f) Find the rocket's acceleration a in S expressed in terms of m_0, dm_0/dt and u, where t is the time in S. Calculate the rocket's rest mass $m_0(t)$ at time t when the radius is R and we set $m_0(0) = M_0$. Find the rocket's restmass $m_0(\theta)$ when it has moved an angle θ from its position P at time $t = 0$ (see Figure 3). What is strange about this result?