Problem Set 9

Problem 9.1

A monocromatic light source is at rest in the laboratory and sends photons with frequncy ν_0 towards a mirror which has its reflective surface perpendicular to the beam direction. The mirror moves away from the light source with velocity v. Use the transformation formulas for 4-momentum and the Planck relation $E = h\nu$ to find the frequency of the reflected light

a) when it is observed in the mirror's rest system.

b) when it is observed in the lab system.

Problem 9.2

To photons in the laboratory system have fequencies ν_1 and ν_2. The angle between the the propagation directions is θ.

a) Write down the expressions for the system's eneregy and momentum in the laboratory system.

b) Find the photons' frequency in the center of mass system.

c) Is it allways possible to find a center of mass system for the photons?

Problem 9.3

We send a photon towards an electron at rest.

a) What is the minimum energy of the photon required for the following process to take place

$$\gamma + e^- \rightarrow e^- + e^- + e^+$$

e^- and e^+ has rest mass m_0.

b) Show that the process

$$\gamma \rightarrow e^- + e^+$$

is impossible.

Problem 9.4

a) Write down the relativistic expressions for a particle's momentum and energy in terms of it's rest mass and velocity. Derive the expressions for velocity and rest mass in terms of momentum and energy.

Figure 1 shows a particle with rest mass m_0 and kinetic energy T. The particle is moving toward another particle at rest with the same rest mass.

b) Find the velocity u of the first particle expressed in terms of $\alpha = T/m_0c^2$.

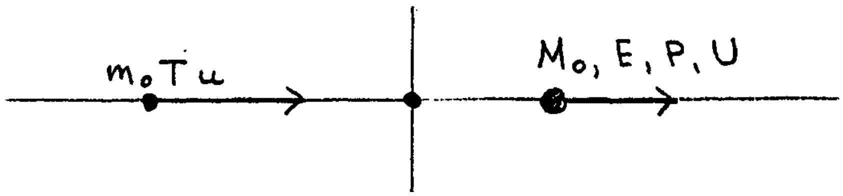

Figure 1: The inelastic collision process in Problem 9.1

First we will assume that the particles collide in such a way that they form one particle after the collision (totally inelastic collision.)

c) Determine the compond particle's energy E, momentum P, velocity U and rest mass M_0. Find the change in the systems kinetic energy due to the collision.

In the rest of the exercise we will assume that the situation before the collision is as described earlier, but that the particles now collide elastic, i.e. both of the the particles rest masses are unchanged after the collision. The collision happens in such a way that the particles after the collision make the same angle, θ, with the x-axis. See Figure 2.

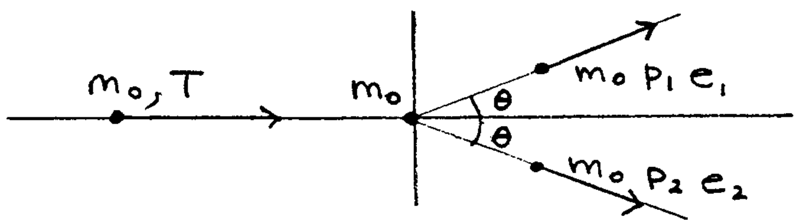

Figure 2: The elastic collision process in Problem 9.1

d) Show that after the collision the particles have the same momentum ($p_1 = p_2$) and energy ($e_1 = e_2$).

e) Determine $e = e_1 = e_2$ and $p = p_1 = p_2$.

f) Determine the angle θ. Find θ in the limiting cases when $\alpha = T/m_0c^2$ goes to zero and infinity. Show that $\theta < \pi/4$.