FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2009

Problem Set 12

Problem 12.1

A thin straight conducting cable, oriented along the z axis in an inertial reference frame S, carries a constant current I. The cable is charge neutral.

a) Show, by use of Ampere's law, that the current produces a rotating magnetic field $\mathbf{B} = B(r)\mathbf{e}_\phi$, where (r, ϕ) are polar coordinates in the x, y plane and $\mathbf{e}_\phi$ is a unit vector in the direction of increasing ϕ. Determine the function $B(r)$.

Consider next the same situation in a reference frame S' that moves with velocity v along the z axis.

b) Use the fact that charge and current densities transform under Lorentz transformation as components of a current 4-vector to show that in S' the conducting cable will be charged. Determine the charge per unit length, λ' and the current I' in this reference frame.

c) Use Gauss' and Ampere's laws to determine the electric and magnetic fields, $\mathbf{E}'$ and $\mathbf{B}'$, as functions of the polar coordinates (r', ϕ') in reference frame S'.

d) Show that if the fields in S' are derived from the fields in S by use of the relativistic transformation formulas for $\mathbf{E}$ and $\mathbf{B}$, that gives the same results as found in c).

Problem 12.2

We consider a monochromatic plane wave that propagates in the z direction in a Cartesian coordinate system. For a given position $\mathbf{r}$ in space the electric field component $\mathbf{E}$ will describe a time dependent, periodic orbit in the x, y plane. The orbit will depend on the form of polarization of the electromagnetic wave.

The electromagnetic wave can generally be viewed as a superposition of two *linearly polarized* waves that propagate in the same direction, and which are polarized in orthogonal directions. We first choose these directions to be defined by the coordinate axes x and y. The amplitudes and the phases of the two partial waves may be different, and the general form of the electric field is therefore

$$\mathbf{E}(\mathbf{r}, t) = a\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi_1)\mathbf{i} + b\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi_2)\mathbf{j} \tag{1}$$

where in general $a \neq b$ and $\phi_1 \neq \phi_2$.

In the following we consider the case where the two partial waves are 90^o out of phase and write this as

$$\mathbf{E}(\mathbf{r}, t) = a\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)\mathbf{i} + b\sin(\mathbf{k}\cdot\mathbf{r} - \omega t)\mathbf{j} \tag{2}$$

a) Show that the orbit described by the time dependent electric field (2) is an ellipse with symmetry axes along the coordinate axes in the x, y plane. What determines the *eccentricity* of the ellipse?

We consider now a different decomposition of the same wave, in linearly polarized components along the rotated directions

$$\mathbf{e}_1 = \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})\,, \quad \mathbf{e}_2 = \frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{j}) \tag{3}$$

b) Show that in this new decomposition, the amplitudes of the two linearly polarized components are equal, $a' = b'$, but the relative phase $\Delta\phi = \phi_1' - \phi_2'$ is different from 90^o (or $\pi/2$ in radians). Show that the relative phase $\Delta\phi$ is determined by the ration a/b in the first decomposition.

c) Assume the amplitude $|\mathbf{E}| = \sqrt{a^2+b^2}$ to be fixed. Plot the orbit of $\mathbf{E}$ with $\mathbf{e}_1$ and $\mathbf{e}_2$ defining the horizontal and vertical axes for a set of different values of the relative phase $\Delta\phi$. Include the cases that correspond to linear and circular polarization and give the values of $\Delta\phi$ for these cases.