FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2009

Problem Set 14

Problem 14.1

The figure shows a straight antenna of length $2a$ lying along the z-axis with its center at the origin. We assume that the charge of the antenna is at all times located at the endpoints. The current in the antenna (between the charged end points) is given by $I = I_0 \sin \omega t$ where ω and I_0 are constants. The antenna is electrical neutral at time $t = 0$. The field point is given by the position vector $\mathbf{r}$ and in spherical coordinates (r, θ, ϕ).

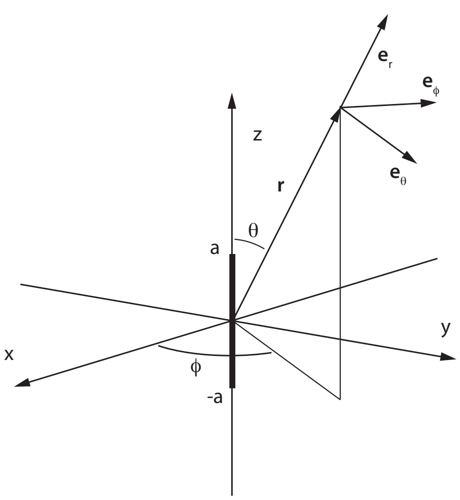

Figure 1:

a) Show that the antenna's electrical dipole moment at time t is given by $\mathbf{p}(t) = \frac{2aI_0}{\omega}(1 - \cos \omega t)\mathbf{k}$, where $\mathbf{k}$ is the unit vector in the z-direction.

We will now assume that the fields can be treated as electrical dipole radiation.

b) Find the components of the $\mathbf{B}$ and $\mathbf{E}$ fields in the directions $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ in the field point (r, θ, ϕ) at time t.

c) Show that the time average of the total radiated power in all directions kan be written as $\langle P \rangle = \frac{RI_0^2}{2}$ and find R (radiation resistance). What is the time average of the total power consumed by the antenna if it has an 'ordinary' resistance R_0 as well?

d) Find R for an antenna of length $2a = 5$ cm which is conducting a current with frequency $f = 150$ MHz. What is the time average of the total radiated power when $I_0 = 30$ A?

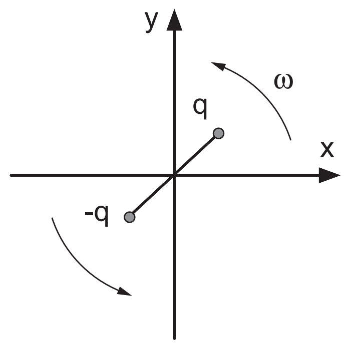

Figure 2:

Problem 14.2 (Exam 2006)

A thin rigid rod of length ℓ rotates in a horizontal plane (the x,y-plane) as shown in Fig. 2. At the two end points there are fixed charges of opposite sign, $+q$ and $-q$. The rod is rotating with constant angular frequency ω. This gives rise to a time dependent electric dipole moment

$$\mathbf{p}(t) = q\ell(\cos\omega t\,\mathbf{i} + \sin\omega t\,\mathbf{j}) \tag{1}$$

a) Use the general expression for the radiation fields of an electric dipole (see the formula collection of the course) to show that the magnetic field in the present case can be written as

$$\mathbf{B}(\mathbf{r},t) = B_0(r)(\cos\theta\sin(\omega(t-\frac{r}{c}))\,\mathbf{i} - \cos\theta\cos(\omega(t-\frac{r}{c}))\mathbf{j} - \sin\theta\sin(\omega(t-\frac{r}{c}) - \phi)\,\mathbf{k} \tag{2}$$

with (r,θ,ϕ) as the polar coordinates of $\mathbf{r}$. Find the expression for $B_0(r)$.

What is the general relation between the electric field $\mathbf{E}(\mathbf{r},t)$ and the magnetic field $\mathbf{B}(\mathbf{r},t)$ in the radiation zone? (A detailed expression for $\mathbf{E}(\mathbf{r},t)$ is not needed.)

b) Show that radiation in the x-direction is linearly polarized. What is the polarization of the radiation in the z-direction?

c) Find the time-averaged expression for the energy density of the radiation. In what direction has the radiated energy its maximum?