

## Problem Set 2

### Problem 2.1

A pendulum consists of a rigid rod, which we consider as massless, and a pendulum bob of mass  $m$ . The point of suspension of the pendulum has horizontal coordinate  $x = s$  and vertical coordinate  $y = 0$ .

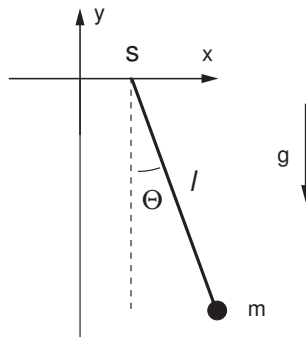


Figure 1:

a) Assume first that the point of suspension is kept fixed, with  $s = 0$ . Use the angle  $\theta$  as generalized coordinate, find the Lagrangian of the system and determine the form of Lagrange's equation for the system. Check that it has the standard form of a pendulum equation.

b) The point of suspension is now released so it can move freely in the horizontal direction ( $x$ -direction). Use  $s$  and  $\theta$  as generalized coordinates for the system and determine the corresponding set of Lagrange's equations. Show that the equations imply that the vertical motion of the pendulum bob is identical to free fall in the gravitational field (in reality restricted by the length  $l$  of the rod).

### Problem 2.2

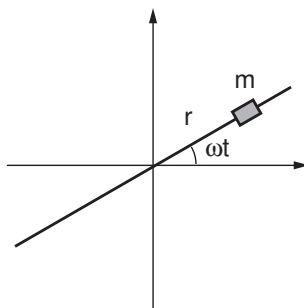


Figure 2:

A small body of mass  $m$  moves without friction on a rod. The rod rotates in the horizontal plane about a fixed point with constant angular velocity  $\omega$ . Find Lagrange's equation for the radial coordinate and solve the equation for the initial condition at  $t = 0$ ,  $\dot{r} = 0$  and  $r = r_0$ .

**Problem 2.3**

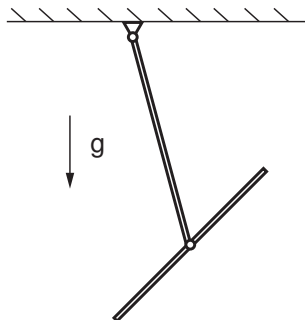


Figure 3: Problem

Two identical rods of mass  $m$  and length  $l$  are connected to each other with a frictionless joint. The first rod is connected to a joint in the ceiling and to a joint at the center of the second rod. Assume that the motion takes place in the vertical plane. Choose suitable generalized coordinates for the system, and find the corresponding Lagrangian. Formulate Lagrange's equations for the system