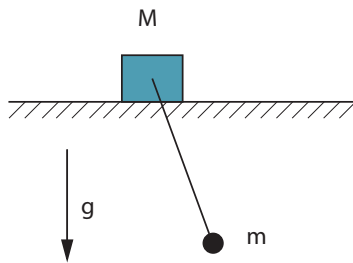


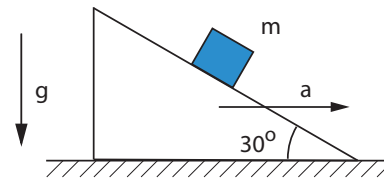
Problem Set 3

Problem 3.1

Choose suitable generalized coordinates for the systems specified below, and find the corresponding Lagrangians. Formulate in each case Lagrange's equations, and interpret the equations, when possible, in terms of other mechanical principles. Search for exact solutions, in the cases where they can be found, and look for possible constants of motion.



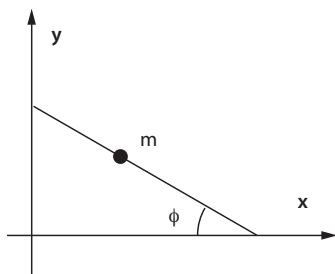
Problem 3.1 a)



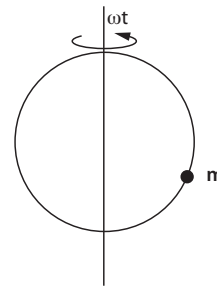
Problem 3.1 b)

a) A pendulum is connected to a box that can slide without friction on a horizontal plane. Assume that the motion takes place in the vertical plane. The pendulum rod is considered as massless.

b) A particle with mass m slides without friction on a tilted plane. The body that constitutes the tilted plane is forced to move horizontally with a constant acceleration a .



Problem 3.1 c)



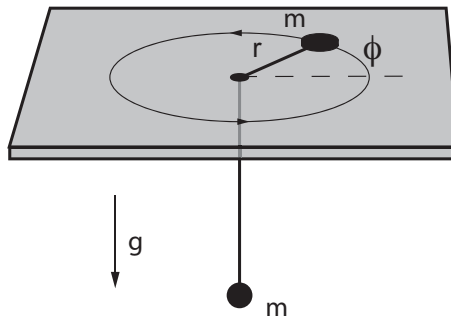
Problem 3.1 d)

c) A rigid rod in the horizontal plane is forced to move in such a way that the end points are in contact with the coordinate axes. The angle φ increases linearly with time. A particle slides without friction along the rod.

d) A rigid circular metal hoop rotates with constant angular velocity around an axis through the center. A particle slides without friction along the circle and there is no gravity.

Problem 3.2

Two bodies with the same mass, m , are connected with a massless rope through a small hole in a smooth horizontal plane. One body is moving on the plane, the other one is hanging at the end of the rope and can move vertically. At all instances the rope is tight. The acceleration due to gravity is g .



a) Find the Lagrange's equations of motion in polar coordinates (r, θ) and explain their physical meaning. Discuss special cases.

b) Reduce the equations of motion to a one dimensional problem in r with an effective potential and discuss the motion.