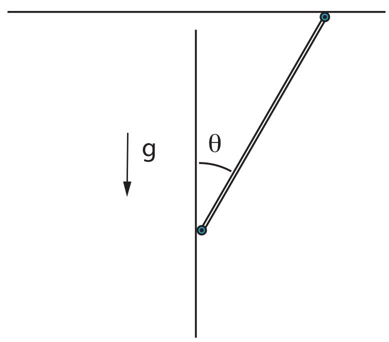
FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2009

Problem Set 4

Problem 4.1

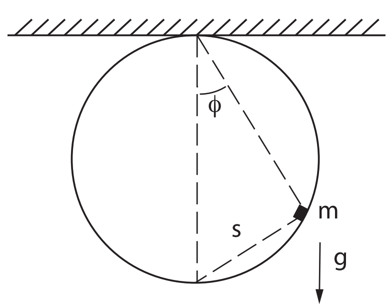
The figure shows a rod of length b and mass m. One endpoint of the rod is constrained to move along a horisontal line and the other endpoint along a vertical line. The two lines are in the same plane. There is no friction and the acceleration due to gravity is g.



a) Find Lagrange's equations with the angle θ as coordinate.
b) Find the period for small oscillations about the equilibrium position.
c) Find the period for oscillations with amplitude $\pi/2$.

Problem 4.2

A particle of mass m is attached to the circumference of a rigid circular hoop of radius r. The the hoop rolls on the underside of a horisontal line.



We assume the hoop to be massless and the motion to take place in a vertical plane. Find the Lagrangian, first with ϕ as generalized coordinate. Find the corresponding Lagrange's equation. Show next that the Lagrangian simplifies to that of a one-dimensional harmonic oscillator when s is used as generalized coordinate. What is the period of oscillations. Why is there a maximal allowed amplitude for the oscillations in s, and what happens when the total energy is larger than the energy corresponding to the maximum amplitude?

Problem 4.3

A particle of mass m and charge q is moving in a magnetic field given by the vectorpotential (in polar coordinates)

$$A_r = A_\theta = 0 \qquad\qquad A_\phi = \frac{k}{r}\tan(\theta/2), \tag{1}$$

where k is a constant. Throughout this problem we will use polar coordinates (r, θ, ϕ) and assume the motion to be non-relativistic. Assume also that there is no gravitational field.

a) Find the corresponding $\mathbf{B}$-field. Do you have a suggestion in what way such a magnetic field can be approximately realized.

b) Find the Lagrangian and Lagrange's equations for the charged particle.

c) Show that the kinetic energy is a constant of motion.

d) Explain the physical meaning of Lagrange's equation for r.

e) Show that there exists solutions of the form

$$r = (a^2t^2 + b^2)^{1/2} \qquad\qquad \theta = \theta_0, \tag{2}$$

where a, b,and θ_0 are constants.

f) Give a physical interpretation of the constants a and b.

g) Make a sketch that shows the magnetic field and a trajectory of the type we have just found.