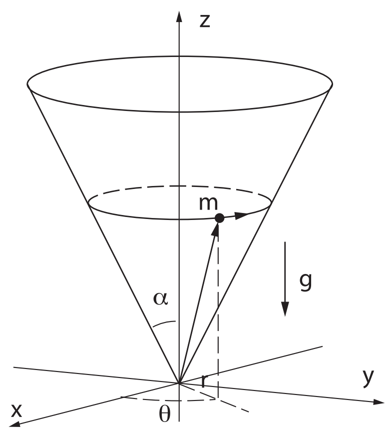
FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2009

Problem Set 5

Problem 5.1

A particle moves on a circular cone of half angle α, which lies symmetrically about the positive z axis, as shown in the figure. The particle has mass m and moves without friction on the inner surface of the cone. The particle's position is given by the polar coordinates (r, ϕ) of the *projection* of the position vector into the x, y plane. The acceleration due to gravity is g in the negative z-direction.



a) Show that the Lagrangian for the particle is

$$L = \frac{1}{2}m(\dot{r}^2(1 + \cot^2 \alpha) + (r\dot{\phi})^2) - mgr \cot \alpha. \quad (1)$$

b) Find Lagrange's equations for the particle.

c) Find two constants of motion and explain their physical meaning.

d) Which initial velocity $\mathbf{v}_0$ must be given to the particle in the point $(r = r_0, \phi = 0)$ to make it move in a horisontal trajectory? Show that the answer can be found from the equations of motion as well as from elementary ideas from Newtonian mechanics?

e) Show that the Hamiltonian is given by

$$H = \frac{p_r^2}{2m(1 + \cot^2 \alpha)} + \frac{p_\phi^2}{2mr^2} + mgr \cot \alpha. \quad (2)$$

f) Show how the equations of motion that was found in part b) also can be found from Hamilton's equations.

g) Show how the two constants of motion in part c) can be found from the Hamiltonian formalism.

Problem 5.2 (Exam 2004 Problem 2)

A small sphere rolls on the inside of a hollow cylinder as shown in the figure. The motion is all the time taking place in a vertical plane (the x, y plane) under the influence of gravity. The inner radius of the cylinder is r an the mass of the sphere is m. The initial velocity of the small sphere is v_0 at the time when the sphere is at the bottom of the cylinder. We assume this velocity to be sufficiently large for the sphere to perform complete circulations inside the cylinder, so that at all times there is contact between the sphere and the cylinder

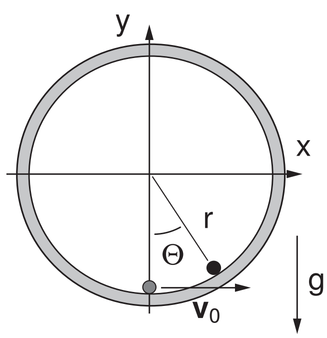


Fig 2a

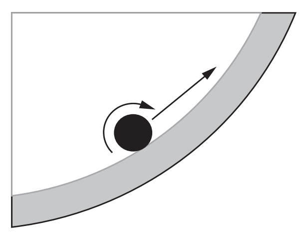


Fig 2b

In the first part of this problem we consider the radius of the sphere to be negligibly small and also disregard the moment of inertia of the sphere about its center of mass (Fig. 2a).

a) Choose a convenient generalized coordinate for the sphere and write the corresponding expression for the Lagrangian.

b) Formulate Lagrange's equation for the generalized coordinate and show that it has the form of a pendulum equation.

c) What is the minimum value for the initial velocity v_0 if the sphere should be in contact with the cylinder under a full revolution? Express this value in terms of r and m.

d) Take into account the effects of the sphere having a small, but finite radius a ($a << r$) and a non-vanishing moment of inertia $I = (2/5)ma^2$ about an axis through the center of mass (Fig. 2b).

What is in that case the correct expression for the Lagrangian, and what is the smallest initial velocity v_0, if the sphere should stay in contact with the cylinder under a full revolution inside of the cylinder?