FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2009

Problem Set 6

Problem 6.1

This problem is an exercise in using the postulates of special relativity in an elementary way.

A railway carriage is moving in a straight line with constant velocity v relative to the earth. The earth is considered as an inertial reference frame S, and in this reference frame the moving carriage has the length L. The situation is shown in Figure 1, where A and B indicate points on the rear wall and front wall of the carriage, respectively. C is a point in the middle of the carriage.

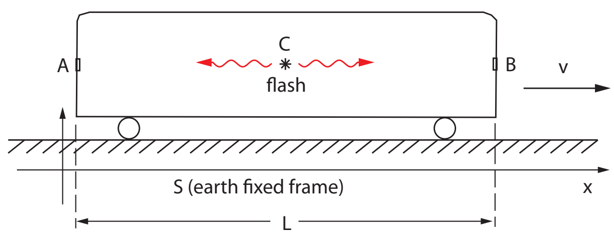


Figure 1:

a) In Figure 2 we have drawn the world line (space-time trajectory) for the midpoint C in a two-dimensional Minkowski diagram of reference frame S. Draw the world lines for the points A and B in the same diagram and show that the angle α between these lines and the time axis is given by $\tan\alpha = v/c$. (Choose the origin of the coordinate system in S so that A has coordinate $x = 0$ at time $t = 0$.)

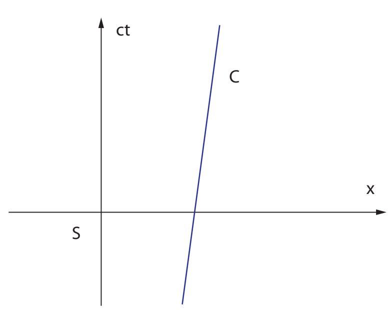


Figure 2:

At a given time t_0 a flash tube is discharged at point C. We will call this event (space-time point) E_0. Some of the light will propagate backwards in the compartment and some will propagate forwards. Let E_1 and E_2 be the events where the light signals hit the rear wall and front wall, respectively. Let us assume that the light is reflected from A and B, and that the two reflected light signals meet at a space-time point E_3.

b) Draw the world lines of the light signals as well as the four events E_0, E_1, E_2 and E_3 in the Minkowski diagram of reference frame S.

c) We introduce the co-moving reference frame S' of the carriage. Explain why E_1 and E_2 are simultaneous in this reference frame and why E_0 and E_3 are at the same point in space in in S'. Is this consistent with the drawing of point b)?

d) Draw the straight line from from E_1 to E_2 in the Minkowski diagram of S and show that the angle between the x-axis and this line is α.

e) Show that if a signal should connect the two space-time points E_1 and E_2 it must have the velocity c^2/v (which is greater than c).

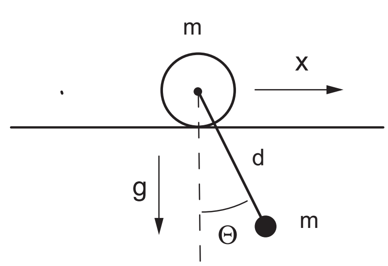
f) Let E be any event, *i.e.*, any space-time point, inside the carriage. Plot in the Minkowski diagram of S the events in the carriage system that are simultaneous with E in the co-moving frame S'. Plot in the same diagram the events that occur at the same place as E in reference frame S'.

Let the coordinates x' and t' of reference frame S' to be chosen such that $x' = 0$, $t' = 0$ corresponds to $x = 0, t = 0$.

g) In the Minkowski diagram of S, the coordinate axes of x and t appear as orthogonal lines. Draw in this diagram the coordinate axes of x' and t', corresponding to $t' = 0$ and $x' = 0$.

h) The lines plotted in g) define non-orthogonal axes for x' and ct'. The space-time position for any event E can in the diagram be read out either as x and ct in the orthogonal coordinate system of S or as x' and ct' in the non-orthognal coordinate system of S'. Explain how.

Problem 6.1. (Midterm Exam 2005)



Figur 1

A composite system is shown in Figure 1. A cylinder with mass m rolls without slipping on a horizontal table. To the axis of the cylinder is attached a pendulum which oscillates freely under the influence of gravity. The pendulum bob has the same mass m as the cylinder and the length of the pendulum rod is d. The rod is considered massless. Choose as generalized coordinates the horizontal displacement x of the cylinder and the angle θ of the pendulum rod relative to the vertical direction. The cylinder has radius R and has a homogeneous distribution of mass. Use the following initial conditions at time $t = 0$, $\dot{x} = 0$, $\dot{\theta} = 0$ and $\theta = \theta_0 \neq 0$.

a) Find the Lagrangian of the composite system.

b) Formulate Lagrange's equations for x og θ. What are the constants of motions that you can identify?

c) Show that by eliminating x that we obtain the following equation of motion for θ,

$$(1 - \frac{2}{5}\cos^2\theta)\ddot{\theta} + \frac{2}{5}\cos\theta\sin\theta\dot{\theta}^2 + \frac{g}{d}\sin\theta = 0 \tag{1}$$

d) Assume $\theta_0 << 1$, so that the pendulum performs small oscillations around $\theta = 0$. Show that in this case the equation of motion reduces to an harmonic oscillator equation and determine the frequency of oscilations.