UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120/FYS 4120 Classical mechanics and electrodynamics
Day of exam: Thursday June 8, 2006
Exam hours: 3 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Øgrim og Lian or Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120/4120

This paper is available also in Norwegian (Bokmål or Nynorsk) language.

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PROBLEM 1
Composite system

A composite mechanical system, shown in Fig. 1, consists of two parts. Part A is a cylinder with

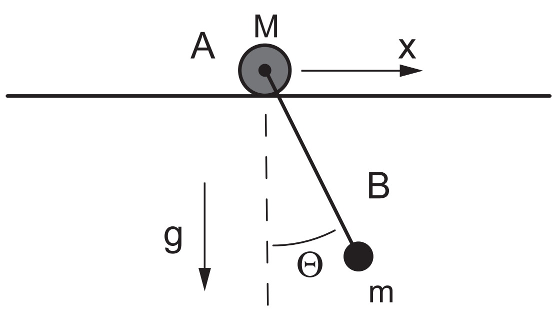

Figure 1:

mass M, radius R and moment of inertia about the symmetry axis $I = \frac{1}{2}MR^2$. The cylinder rolls without sliding on a horizontal plane. Part B is a pendulum with the pendulum rod attached to the symmetry axis of the cylinder. It oscillates without friction about the point of attachment in the plane orthogonal to the symmetry axis of the cylinder. The pendulum rod has length L and we consider it as massless. The mass of the pendulum bob is $m = M/2$.

Use in the following the horizontal displacement x of the cylinder and the pendulum angle θ as generalized coordinates for the composite system.

a) Find the Lagrangian of the system expressed as a function of the generalized coordinates and their time derivatives.

b) Examine if there are cyclic coordinates, and find constants of motion.

c) Formulate Lagrange's equations for the system and assume the following initial conditions: $x = 0$, $\dot{x} = 0$, $\theta = \theta_0$, $\dot{\theta} = 0$ at time $t = 0$. Assume $\theta_0 << 1$ and simplify the equations by using a small angle approximation. Show that the system in this case will perform small oscillations of the form $\theta(t) = \theta_0 \cos \omega t$ and determine the oscillation frequency ω. What is the corresponding expression for $x(t)$?

PROBLEM 2
Accelerated charge

An electron, with charge e, moves in a constant electric field $\mathbf{E}$. The motion is determined by the relativistic Newton's equation

$$\frac{d}{dt}\mathbf{p} = e\mathbf{E} \tag{1}$$

where $\mathbf{p}$ denotes the relativistic momentum $\mathbf{p} = m_e \gamma \mathbf{v}$, with m_e as the electron rest mass, $\mathbf{v}$ as the velocity and $\gamma = 1/\sqrt{1-(v/c)^2}$ as the relativistic gamma factor. We assume the electron to move along the field lines, that is, there is no velocity component orthogonal to $\mathbf{E}$.

a) Show that the electron has a constant proper acceleration $\mathbf{a}_0 = e\mathbf{E}/m_e$, which is the acceleration in an instantaneous rest frame of the electron.

b) Show that if $v = 0$ at time $t = 0$, then γ depends on time t as

$$\gamma = \sqrt{1 + \kappa^2 t^2} \tag{2}$$

and find κ expressed in terms of a_0.

c) Show that if we write $\gamma = \cosh \kappa\tau$ then τ is the proper time of the electron.

As a reminder we give the following functional relations:

$$\cosh^2 x - \sinh^2 x = 1\,, \quad \frac{d}{dx}\cosh x = \sinh x\,, \quad \frac{d}{dx}\sinh x = \cosh x \tag{3}$$

PROBLEM 3
Rotating dipole

A thin rigid rod of length ℓ rotates in a horizontal plane (the x,y-plane) as shown in Fig. 2. At the two end points there are fixed charges of opposite sign, $+q$ and $-q$. The rod is rotating with constant angular frequency ω. This gives rise to a time dependent electric dipole moment

$$\mathbf{p}(t) = q\ell(\cos\omega t\,\mathbf{i} + \sin\omega t\,\mathbf{j}) \tag{4}$$

a) Use the general expression for the radiation fields of an electric dipole (see the formula collection of the course) to show that the magnetic field in the present case can be written as

$$\mathbf{B}(\mathbf{r},t) = B_0(r)(\cos\theta\sin(\omega(t-\frac{r}{c}))\,\mathbf{i} - \cos\theta\cos(\omega(t-\frac{r}{c}))\mathbf{j} - \sin\theta\sin(\omega(t-\frac{r}{c}) - \phi)\,\mathbf{k} \tag{5}$$

with (r,θ,ϕ) as the polar coordinates of $\mathbf{r}$. Find the expression for $B_0(r)$.

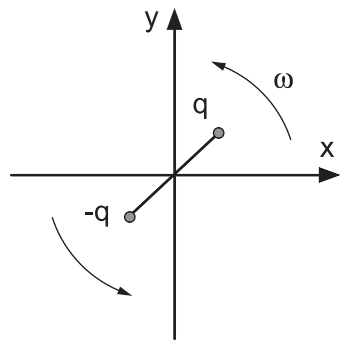

Figure 2:

What is the general relation between the electric field $\mathbf{E}(\mathbf{r}, t)$ and the magnetic field $\mathbf{B}(\mathbf{r}, t)$ in the radiation zone? (A detailed expression for $\mathbf{E}(\mathbf{r}, t)$ is not needed.)

b) Show that radiation in the x-direction is linearly polarized. What is the polarization of the radiation in the z-direction?

c) Find the time-averaged expression for the energy density of the radiation. In what direction has the radiated energy its maximum?

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Det matematisk-naturvitenskapelige fakultet

Eksamen i: FYS 3120/FYS 4120 Klassisk mekanikk og elektrodynamikk
Eksamensdag: Mandag 4. juni 2007
Tid for eksamen: kl. 14:30 (3timer)
Oppgavesettet er på 3 sider
Tillatte hjelpemidler: Godkjent kalkulator
Øgrim og Lian eller Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120/4120

Kontroller at oppgavesettet er komplett før du begynner å besvare spørsmålene.

OPPGAVE 1
To sammenbundne legemer

Et mekanisk system er sammensatt av en liten kloss og en kule forbundet med en snor. Klossen

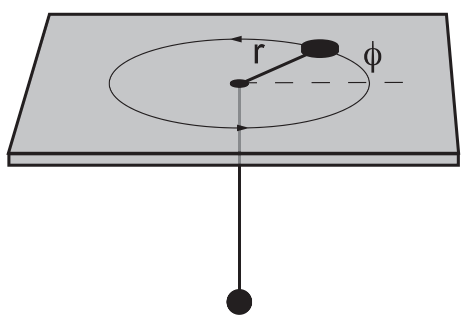

Figure 1:

kan gli friksjonsløst på et horisontalt bord. Snoren er ført langs bordet gjennom et hull til kula som bare beveger seg vertikalt, slik det er vist i Fig. 1. Massen til klossen er $2m$ med m som kulas masse. Vi regner snora som masseløs og uelastisk og at den hele tiden er strukket. Den har lengde d. Klossen regner vi som tilstrekkelig liten til at treghetsmomentet om massemiddelpunktet er neglisjerbart.

a) Benytt polarkoordinatene (r, ϕ) til klossen på bordet som generaliserte koordinater og vis at Lagrangefunksjonen til det sammensatte systemet har formen

$$L = \frac{3}{2}m\dot{r}^2 + mr^2\dot{\phi}^2 + mg(d-r) \tag{1}$$

b) Sett opp Lagranges ligninger. Hva menes med at vinkelkoordinaten er syklisk? Sett opp uttrykket for den tilhørende bevegelseskonstant l, og benytt det til å redusere bevegelsesligningene til én ligning, i den radielle variabelen r. Hvilken fysisk tolkning har l?

c) Vis v.h.a. den radielle ligningen at det finnes en stabil situasjon hvor klossen går i sirkelbane med konstant radius r_0. Anta at r avviker litt fra r_0, $r = r_0 + \rho$ med $|\rho| << r_0$. Vis at den radielle bevegelsen blir små svingninger om r_0, mens klossen sirkulerer i planet. Hva er frekvensen for de små oscillasjonene om r_0?

OPPGAVE 2
Sirkulerende elektron

Et elektron går i sirkelbane i et konstant magnetfelt (vinkelrett på banen) i en syklotron. Radius i banen er $R = 10m$ og den relativistiske gammafaktoren til elektronet er $\gamma = 100$. Massen til elektronet er $m_e = 9.1 \times 10^{-31} kg$, elektronladningen er $e = -1.6 \times 10^{-19} C$ og lyshastigheten $c = 3.0 \cdot 10^8 m/s$.

a) Hvor stor er elektronets fart uttrykt ved lyshastigheten. Finn også vinkelhastigheten ω og akselerasjonen a til elektronet målt i laboratoriesystemet (dvs. i inertialsystemet hvor syklotronen er i ro).

Benytt i det følgende koordinater hvor sirkelbanen til elektronet ligger i x,y-planet med sentrum av sirkelen i origo. Anta at magnetfeltet er rettet langs den positive z-aksen.

b) Sett opp uttrykkene for elektronets 4-vektorkoordinater x^μ ($\mu = 0, 1, 2, 3$), som funksjon av R, ω, γ og egentiden τ. Bestem også 4-hastigheten og 4-akselerasjonen. Hvor stor er egenakselerasjonen a_0 (akselerasjonen målt i det momentane hvilesystemet) uttrykt ved a?

c) Anta vi studerer bevegelsen i elektronets momentane hvilesystem. Hva er feltstyrken til magnetfeltet $\mathbf{B}'$ og til det elektriske feltet $\mathbf{E}'$ i dette referansesystemet uttrykt ved magnetfeltet $\mathbf{B}$ og elektronhastigheten $\mathbf{v}$ i laboratoriesystemet? (Benytt de generelle uttrykk for Lorentztransformasjon av elektromagnetiske felter.) Sjekk at bevegelsesligningen er oppfylt i det momentane hvilesystemet når den er oppfylt i labsystemet, ved å benytte de uttrykkene som er funnet for egenakselerasjonen og for de transformerte feltene. (Vær oppmerksom på at på vektorform peker akselerasjonene $\mathbf{a}$ og $\mathbf{a}_0$ i de to inertialsystemene i samme retning.)

OPPGAVE 3
Oscillerende strøm

I en sirkelformet strømsløyfe med radius a går det en oscillerende strøm på formen $I = I_0 \cos \omega t$. Strømsløyfen ligger i x,y-planet. Vi benytter betegnelsene $\mathbf{e}_x$, $\mathbf{e}_y$ og $\mathbf{e}_z$ for de kartesiske enhetsvektorene i x, y- og z-retningene, for å kunne reservere $\mathbf{j}$ for strømtettheten. Strømsløyfen regnes å være ladningsnøytral.

a) Forklar hvorfor det elektriske dipolmomentet $\mathbf{p}$ til strømsløyfen er lik null, og vis at det magnetiske momentet har tidsavhengigheten $\mathbf{m}(t) = m_0 \cos \omega t \, \mathbf{e}_z$, med m_0 som en konstant. Bestem m_0 uttrykt ved a og I_0.

Vi minner om de generelle uttrykkene for strålingsfeltet fra en magnetisk dipol,

$$\mathbf{E}(\mathbf{r}, t) = -\frac{\mu_0}{4\pi c r} \ddot{\mathbf{m}}_{ret} \times \mathbf{n} \, ; , \quad \mathbf{B}(\mathbf{r}, t) = -\frac{1}{c} \mathbf{E}(\mathbf{r}, t) \times \mathbf{n} \tag{2}$$

hvor $\mathbf{m}_{ret} = \mathbf{m}(t - r/c)$ og $\mathbf{n} = \mathbf{r}/r$. Anta i det følgende at vi studerer feltene langt fra strømsløyfen (i strålingssonen) hvor uttrykkene (2) er gyldige.

b) Sett opp uttrykket for strålingsfeltene for punkter på x-aksen langt fra kilden, og vis at de har form av elektromagnetiske bølger som forplanter seg i x-retningen bort fra strømsløyfen. Hva slags polarisasjon har bølgene?

c) Benytt det generelle uttrykket for Poyntings vektor $\mathbf{S}$ til å finne den utstrålte effekt pr. romvinkelenhet, $\frac{dP}{d\Omega}$, i x-retningen. Hvor stor er den utstrålte effekt i z-retningen?

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Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Tuesday June 3, 2008
Exam hours: 3 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Øgrim og Lian or Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120
Language: The solutions may be written in Norwegian or English depending on your own preference.

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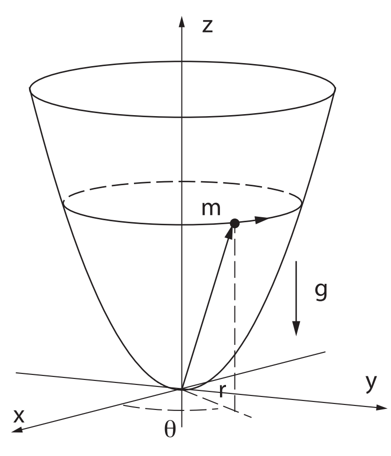
PROBLEM 1
Particle on a constrained surface
A particle moves on a parabolic surface given by the equation $z = (\lambda/2)(x^2 + y^2)$ where z is the Cartesian coordinate in the vertical direction, x and y are orthogonal coordinates in the horizontal plane and λ is a constant. The particle has mass m and moves without friction on the surface under influence of gravitation. The gravitational acceleration g acts in the negative z-direction. The particle's position is given by the polar coordinates (r, θ) of the *projection* of the position vector into the x, y plane.



a) Show that the Lagrangian for this system is

$$L = \frac{1}{2}m[(1 + \lambda^2 r^2)\dot{r}^2 + r^2\dot{\theta}^2 - g\lambda r^2] \tag{1}$$

and find Lagrange's equations for the particle.

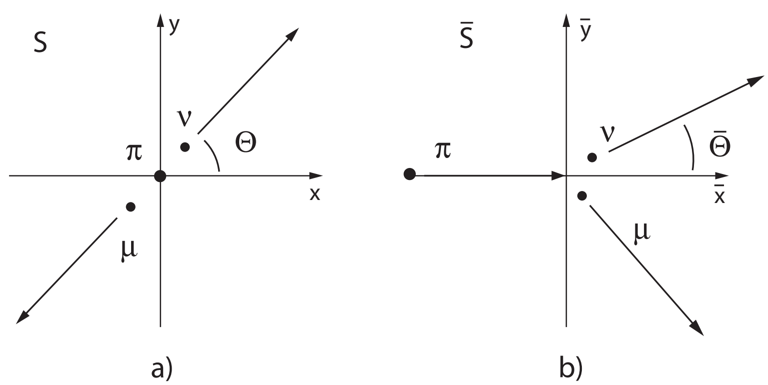
b) Use the fact that there is a cyclic coordinate to show that the equations can be reduced to a single equation in the radial variable r. What is the condition for the particle to move in a circle with radius $r = r_0$?

c) Assume that the path of the particle deviates little from the circular motion so that $r = r_0 + \rho$, where ρ is small. Show that under this condition the radial equation can be reduced to a harmonic oscillator equation for the small variable ρ and determine the corresponding frequency. Give a qualitative description of the motion of the particle.

PROBLEM 2

Particle decay

Pi-mesons (pions) are unstable elementary particles. We consider here a decay process of a charged pion π^+ into a muon μ^+ and a neutrino ν_μ. The masses of the particles are $m_\pi = 273 m_e$ and $m_\mu = 207 m_e$, with $m_e = 0.51 MeV/c^2$ as the electron (rest) mass. (The standard energy unit in particle physics, eV = electron volt is used. The speed of light is as usual represented by c.) The mass of the neutrino is so small that the particle can be regarded as massless.



In the figure the decay process is shown both in the rest frame S of the pion, and in the laboratory frame $\bar{S}$. In this frame the pion moves with the velocity $v = (4/5)c$ along the x axis. To distinguish the variables of the two reference frames S and $\bar{S}$ we mark the variables of the latter with a "bar", so that for example the angle of the neutrino relative to the x axis in S is θ and the corresponding angle in $\bar{S}$ is $\bar{\theta}$.

a) We study first the process in the rest frame S. Set up the equations for conservation of relativistic energy and momentum and use them to determine the energy and (the absolute value of) the momentum of the muon and of the neutrino in this reference system. (Use MeV as unit for energy and MeV/c as unit for momentum.)

b) Use the transformation formula for relativistic 4-momenta to determine the energy of the muon and of the neutrino in the lab frame $\bar{S}$.

c) In the rest frame S all directions for the neutrino momentum are equally probable. Show that this means that in the lab frame $\bar{S}$ the probability is 0.5 for finding the neutrino in a direction with angle $\bar{\theta} < 36.9^o$.

PROBLEM 3
Electric dipole radiation
An electron (with charge e and mass m) is moving with constant speed in a circle under the influence of a constant magnetic field $\mathbf{B}_0$. The magnetic field is directed along the z axis while the motion of the electron takes place in the x, y plane. We assume the motion of the electron to be non-relativistic.

Since the electron is accelerated it will radiate electromagnetic energy and thereby loose kinetic energy when no energy is added to the particle.

a) By use of Larmor's radiation formula, find an expression for the radiated energy per unit time expressed in terms of the radius r of the electron orbit and the cyclotron frequency $\omega = -eB_0/m$.

b) Show that the radius of the electron orbit is slowly reduced with an exponential form of the time dependence, $r = r_0\, e^{-\lambda t}$, and determine λ.

c) The electromagnetic fields produced by the moving charge are essentially electric dipole radiation fields. What is the electric dipole moment of the circulating electron? Give the expressions for the radiation fields $\mathbf{E}(z, t)$ and $\mathbf{B}(z, t)$ on the z axis far from the electron. Show that they correspond to a propagating wave, with direction away from the electron, and determine the form of polarization of the wave.

Expressions found in the formula collection of the course may be useful for this problem.

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Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Tuesday June 2, 2009
Exam hours: 3 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Øgrim og Lian or Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120
Language: The solutions may be written in Norwegian or English depending on your own preference.

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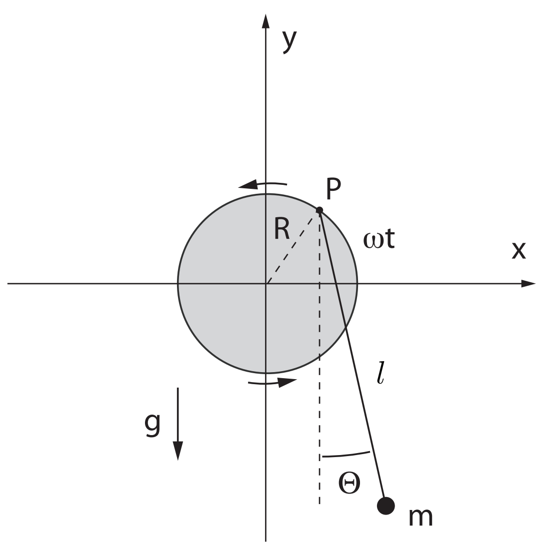
PROBLEM 1
Pendulum attached to a rotating disk
A pendulum is attached to a circular disk of radius R, as illustrated in Fig. 1. The end of the pendulum rod is fixed at a point P on the circumference of disk. The disk is vertically oriented and it rotates with a constant angular velocity ω. The pendulum consists of a rigid rod of length l which we consider as massless and a pendulum bob of mass m. The pendulum oscillates freely about the point P under the influence of gravity.



a) Show that the Lagrangian for this system, when using as variable the angle θ of the pendulum rod relative to the vertical direction, has the form

$$L = m[\frac{1}{2}l^2\dot{\theta}^2 + lR\omega\sin(\theta - \omega t)\dot{\theta} + gl\cos\theta + \frac{1}{2}R^2\omega^2 - gR\sin\omega t] \quad (1)$$

b) Formulate Lagrange's equation for the system and write it as a differential equation for θ.

For $\omega = 0$ the equation reduces to a standard pendulum equation. Assume in the following ω to be non-vanishing, but sufficiently small so the ω-dependent contribution to the equation of motion can be viewed as a small periodic perturbation to the pendulum equation. In that case there are solutions corresponding to small oscillations, $|\theta| << 1$, which are modified by the perturbation.

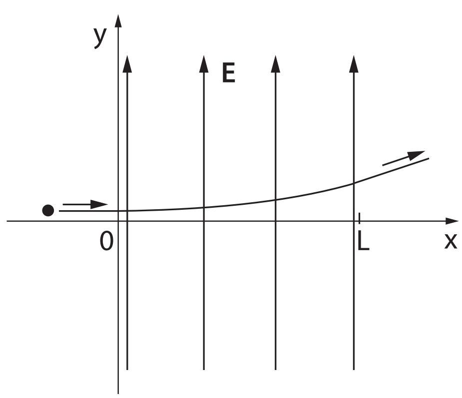
c) Show that under assumption that $|\theta|$ and ω are sufficiently small the equation of motion for the pendulum can be approximated by the equation for a *driven* harmonic oscillator, subject to a periodic force. Show that it has a solution of the form $\theta(t) = \theta_o \cos\omega t$ and determine the amplitude θ_0 in terms of the parameters of the problem.

Based on this solution can you give a more precise meaning to the phrase "sufficiently small ω" as the condition for $\theta_o \cos\omega t$ to be a good approximation to a solution of the full equation of motion?

PROBLEM 2

Charged particle in a constant electric field

A particle with charge q and rest mass m moves with relativistic speed through a region $0 < x < L$ where a constant electric field $\mathbf{E}$ is directed along the y-axis, as indicated in the figure. The particle enters the field at $x = 0$ with momentum $\mathbf{p}_0$ in the direction orthogonal to the field. The relativistic energy at this point is denoted $\mathcal{E}_0$. (Note that we write the energy as $\mathcal{E}$ to avoid confusion with the electric field strength E.)



a) Use the equation of motion for a charged particle in an electric field to determine the time dependent momentum $\mathbf{p}(t)$ and relativistic energy $\mathcal{E}(t)$ (without the potential energy) of the particle inside in the electric field. What is the relativistic gamma factor $\gamma(t)$ expressed as a function of coordinate time t?

b) Find the velocity components $v_x(t)$ and $v_y(t)$ and explain the relativistic effect that the velocity in the x-direction decreases with time even if there is no force acting in this direction.

c) Show that the proper time $\Delta\tau$ spent by the particle on the transit through the region $0 < x < L$ is proportional to the length L, $\Delta\tau = \alpha L$, and determine α.

d) What is the transit time Δt through the region when measured in coordinate time?

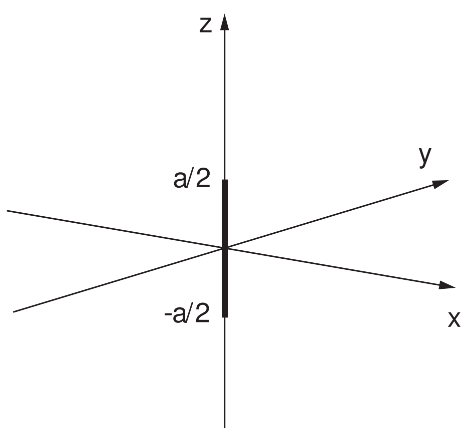
We remind about the integration formula $\int dx \frac{1}{\sqrt{1+x^2}} = \operatorname{arc\,sinh} x + C$.

PROBLEM 3
Radiation from a linear antenna

A so-called *half-wave center-fed* antenna is formed by a thin linear conductor of length a. It is oriented along the z-axis as shown in the figure. An alternating current is running in the antenna, of the form

$$I(z,t) = I_0 \cos\frac{\pi z}{a} \cos\omega t\ , \quad -a/2 < z < a/2 \tag{2}$$

In the following $\lambda(z,t)$ denotes the linear charge density of the antenna (charge per unit length). At time $t = 0$ the antenna is charge neutral, so that $\lambda(z,0) = 0$.



a) Show that the charge density and current satisfy the relation

$$\frac{\partial\lambda}{\partial t} + \frac{\partial I}{\partial z} = 0 \tag{3}$$

and find λ as a function of z and t.

b) Show that the electric dipole moment of the antenna has the form

$$\mathbf{p}(t) = p_0 \sin\omega t\ \mathbf{k} \tag{4}$$

with $\mathbf{k}$ as the unit vector along the z-axis, and determine the constant p_0.

c) Use the expressions for electric dipole radiation to determine the electric and magnetic fields in a point at a large distance r from the antenna on the x-axis. What is the type of polarization of the radiation from the antenna in this direction?

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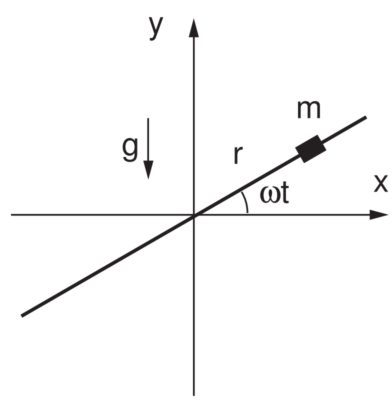
Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Tuesday June 4, 2013
Exam hours: 4 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Angell/Øgrim og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120
Language: The solutions may be written in Norwegian or English depending on your own preference. En kort ordliste finnes etter oppgavene.

Make sure that your copy of this examination paper is complete before answering .

PROBLEM 1
Constrained motion
A small body with mass m is constrained to move on straight line, which rotates with constant angular velocity in a vertical plane, here identified as the xy-plane. The y-axis is pointing in the vertical direction, as shown in the figure. The point of rotation, which lies on the line, is chosen as origin. At $t = 0$ the rotating line coincides with the x-axis. Friction is assumed to be negligible, and the only applied force acting on the body is the gravitational force mg. In the following the radial coordinate r is chosen as generalized coordinate, and the body is treated as pointlike.



a) Find the Lagrangian for this system, expressed in terms of r, $\dot{r}$ and t. Derive the corresponding Lagrange's equation.

b) Show that the equation has a particular solution of the form $r = k \sin \omega t$ and determine the constant k. Show that the solution corresponds to circular motion with constant speed, with the circle centered at the point $(x, y) = (0, k/2)$. What is the speed of the particle and the radius of the circle?

c) Find the general solution of the equation of motion.

PROBLEM 2
Hyperbolic space-time motion

A straight rod is moving along the x-axis of an inertial reference frame S. The two endpoints A and B follow hyperbolic space-time trajectories, described the following time dependent x-coordinates in S,

$$x_A = c\sqrt{t^2 + c^2/a^2}\,, \quad x_B = c\sqrt{t^2 + c^2/b^2} \tag{1}$$

c is the speed of light, and a and b are positive constants, with $b < a$.

a) A second inertial frame S' moves along the x-axis with velocity v relative to S. The coordinates of the two reference frames are chosen to coincide at the space-time point $x = t = 0$.

Show that the motion of A and B, when expressed in terms of the coordinates of S', has precisely the same form as in S,

$$x'_A = c\sqrt{t'^2 + c^2/a^2}\,, \quad x'_B = c\sqrt{t'^2 + c^2/b^2} \tag{2}$$

(To demonstrate this it may be convenient to rewrite the above relations in terms of the squared coordinates x^2 and t^2.)

b) At time $t = 0$ the frame S is an instantaneous rest frame of both A and B. Show this and find the distance between A and B measured in S at this moment. The same results are valid for the reference frame S' at time $t' = 0$.

Based on this we may conclude that for any point on the space-time trajectory of A, the instantaneous inertial rest frame of A is a rest frame also for B. Furthermore the distance between A and B, when measured in the instantaneous inertial rest frame, is constant. Explain these conclusions.

c) Use the above results to show that the proper accelerations of the A and B are constants, and give the values of these.

d) At a given instant $t = 0$ a light signal with frequency ν_0 is sent from A and is subsequently received at B. What is the velocity of B (measured in S) when the signal is received, and what is the frequency of the signal, measured at B? (To answer the last question it may be convenient to use the relation between frequency and four-momentum for a photon sent from A to B.)

PROBLEM 3
Radiation from a current loop

In a circular loop of radius a an oscillating current of the form $I = I_0 \cos \omega t$ is running. The current loop lies in the x, y-plane, with the center of the loop at the origin. The loop is at all times charge neutral. We assume $a\,\omega << c$.

a) Show that the magnetic dipole moment has the following time dependence, $\mathbf{m}(t) = m_0 \cos \omega t\, \mathbf{e}_z$, with m_0 as a constant and $\mathbf{e}_z$ as a unit vector in the z-direction. Find m_0 expressed in terms of a and I_0. Far from the current loop, the fields are dominated by the magnetic dipole radiation field. Explain why.

b) The magnetic dipole radiation fields have the general form

$$\mathbf{E}(\mathbf{r}, t) = -\frac{\mu_0}{4\pi c r} \ddot{\mathbf{m}}_{ret} \times \mathbf{n}\,;, \quad \mathbf{B}(\mathbf{r}, t) = -\frac{1}{c} \mathbf{E}(\mathbf{r}, t) \times \mathbf{n} \tag{3}$$

with $\mathbf{m}_{ret} = \mathbf{m}(t - r/c)$, $\mathbf{n} = \mathbf{r}/r$, and $r >> a$. For the present case give the full expressions of the fields, as functions of r and t, and written in terms of the orthonormalized vectors $\{\mathbf{n} = \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi\}$ of the polar coordinate system. Do they form, as expected, waves that propagate in the radial direction away from the loop? Explain. Characterize the polarization of the waves.

c) Use the general expression for Poynting's vector $\mathbf{S}$ to find the radiated power per unit solid angle $\frac{dP}{d\Omega}$, expressed as a function of the polar angle θ (angle relative to the z-axis). Find the total radiated power, integrated over all directions and averaged over time.

d) A second conducting loop, identical to the first one, is placed at a large distance r from the first loop, with the center of the loop in the x, y-plane. It is used as a receiver, with the radiation from the first loop inducing a current in the second loop. Let $\mathbf{u}$ be a unit vector orthogonal to the plane of the second loop. In what direction should $\mathbf{u}$ be oriented for the second loop to receive the maximal signal?

ORDLISTE

engelsk	**norsk**
Lagrangian	Lagrangefunksjon
angular velocity	vinkelhastighet
instantaneous rest frame	momentant hvilesystem
proper acceleration	egenakselerasjon
current loop	strømsløyfe
radiated power	utstrålt effekt