FYS3120 Classical mechanics and electrodynamics
Mid-term exam – Spring term 2017

Your **candidate number**

March 26, 2017

Important information:

- Your answers are to be submitted electronically as pdf-files, either generated from LaTeX or scanned, using the devilry submission system, at the latest Friday 31st of March at 16.00 local time (GMT+1).

- This deadline is absolute.

- You **must** clearly mark your answer with the candidate number assigned to you for this course, as the answers are to be kept anonymous during grading. You can find this number on Studentweb. Any submission without a candidate number will result in an automatic fail. We would strongly prefer (as in, it would really annoy us if you did not do this) if the pdf was named `XX.pdf`, where `XX` is your candidate number.

- This mid-term exam counts for roughly 25% of the total grade in FYS3120, and you must receive a passing score on the mid-term in order to pass the course.

- As this is a home-exam you are free to use any sources of information you may want, and you may collaborate with other students on solving the problems. However, the text of the submitted answers must be your own, and the usual rules of plagiarism apply. (We may check answers for similarities.)

- The best possible score on this exam is 25 points. Up to one point will be given for clear, concise and well presented answers, including appropriate figures and/or diagrams.

- You may give your answers either in English or Norwegian.

- Good luck!

Question 1 A (boring) Lagrangian

We will begin by considering a free non-relativistic particle (no potential) of mass m moving in three dimensions with no constraints.

a) Pick a sensible coordinate system for this problem and write down the Lagrangian. [1 point]

b) Find the canonical, or conjugate, momenta for the coordinates and compare to the regular (mechanical) momentum. [1 point]

c) Find the cyclic coordinates. [0.5 point]

d) What are the conserved quantities / constants of motion in this problem?[1] [0.5 point per quantity, max 3 points]

We will now repeat the above using instead a relativistic description of the same free particle.

e) Write down a Lagrangian for the corresponding relativistic case that is invariant under Lorentz transformations and demonstrate that it is indeed invariant. [1.5 points]

f) Find the conserved quantities / constants of motion for this Lagrangian. [0.5 point per quantity, max 2 points]

g) We will now look at a small Lorentz transformation where the Lorentz transformation tensor is given as

$$L^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}. \tag{1}$$

Here $\delta^{\mu}{}_{\nu}$ is the Kronecker delta and $\omega^{\mu}{}_{\nu}$ is an infinitesimal parameter, meaning we can disregard higher orders of ω. Show that $\omega^{\mu}{}_{\nu}$ must be antisymmetric, *i.e.* that $\omega^{\mu}{}_{\nu} = -\omega_{\nu}{}^{\mu}$. [1 point]

h) Assume that a small Lorentz transformation between two reference frames changes the path $x^{\mu}(\tau)$ of a particle according to

$$\delta x^{\mu}(\tau) = x'^{\mu}(\tau) - x^{\mu}(\tau) = \omega^{\mu}{}_{\nu} x^{\nu}(\tau). \tag{2}$$

Show that the corresponding change in the Lagrangian is

$$\delta L = \left(\frac{\partial L}{\partial x^{\mu}} x^{\nu} + \frac{\partial L}{\partial U^{\mu}} U^{\nu} \right) \omega^{\mu}{}_{\nu}. \tag{3}$$

[1 point]

[1] The different components of a vector counts as separate quantities.

i) Use Lagrange's equation to show that you can also write Eq. (3) as

$$\delta L = \frac{1}{2}\omega_{\nu\mu}\frac{d}{d\tau}\left(x^{\mu}\frac{\partial L}{\partial U_{\nu}} - x^{\nu}\frac{\partial L}{\partial U_{\mu}}\right). \tag{4}$$

[1.5 points]

j) Identify which quantities are conserved because of the invariance under Lorentz transformations. [1.5 points]

Question 2 Relativistics

Two particles with mass m and a photon is sent out from a source at the same time and in the positive x-direction of the rest frame S of the source. The massive particles are moving with constant velocity v_1 and $v_2 > v_1$ in this frame. Draw a Minkowski-diagram of the motion of the source, both massive particles and the photon in S, and draw the axis of the rest frame S' of the slowest particle. Show that the difference in rapidity of the two massive particles is the same in S and S'.[2] [5 points]

Question 3 Finding the shortest way

Using calculus of variations, find the shortest path between two points on a sphere. We want the answer in terms of a function $\phi(\theta)$ using spherical coordinates (r, θ, ϕ). By the rotational symmetry of the problem you may assume that the starting point is $(r, \frac{\pi}{2}, 0)$. To simplify the answer you may also ignore the special solutions through this point with constant $\phi = 0$ or $\theta = \frac{\pi}{2}$. [5 points]

[2] In fact this is always true, rapidity differences are unchanged by boosts no matter which reference frames you look at. I am sure you can see this from your proof!