UiO : Fysisk institutt

Det matematisk-naturvitenskapelige fakultet

Lecture 10

This week

- **Monday:** Relativity: fundamental principles. (Sections 4.1 and 4.2)
- **Wednesday:** Relativistic four-vectors, Lorentz transformations and Minkowski diagrams. (Sections 4.3, 4.4, and 4.5)
- **Problem session:** No problem session this week. Problem set 5 is up on the web-page, to be handed in Monday 26th of February. Includes mid-term exam question from 2008 (famous physics problem).

Recap

- **Hamilton's principle** or **the principle of least action** says that the action

$$S[q(t)] = \int_{t_1}^{t_2} L(q(t), \dot{q}(t), t)\, dt$$

as a function of the path q(t) is unchanged for small variations

$$q(t) \rightarrow q(t) + \delta q \quad \text{with} \quad \delta q(t_1) = \delta q(t_2) = 0$$

around the trajectory that fulfils the e.o.m., i.e.

$$\delta S = 0$$

Recap

- This can be shown to be equivalent to Lagrange's equations through

$$\delta S = \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} \right) \delta q \; dt = 0$$

- The derivation of this equation assumes nothing about mechanics and can be used to solve minimization problems for parametrized functions q(t) (**calculus of variations**), as long as the integral (functional) to be minimized depends on q and qdot.

Plan for today

- Relativity: fundamental principles. (Sections 4.1 and 4.2)
  - Some basic concepts
    - Galilean transformations
    - Lorentz transformations
    - Rapidity
  - Invariant distance & the metric
  - A first look at four-vectors for space-time (if time)

Summary

- To have the same velocity of light in all frames we introduce Lorentz transformations

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t = \gamma\left(t - \frac{v}{c^2} x\right)$$

- We define an invariant distance (metric)

$$\Delta s^2 \equiv c^2 \Delta t^2 - \Delta x^2$$

which is the same in all reference frames.