UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 12

This week

- **Monday:** Length contraction, time dilation, proper time and the twin-paradox (Sections 5.1-5.4)
- **Wednesday:** Relativistic four-vectors and the Lorentz transformations, general four-vectors. (Sections 6.1-6.4)
- **Problem session:** Problem set 5, last set focused on analytical mechanics.
- Additional problems available for Part I of the course.

Recap

- We can write **Lorentz transformations** as the matrix multiplication (note index system!)

$$x'^{\mu} = L^{\mu}{}_{\nu} x^{\nu}$$

where, for a boost in the x-direction,

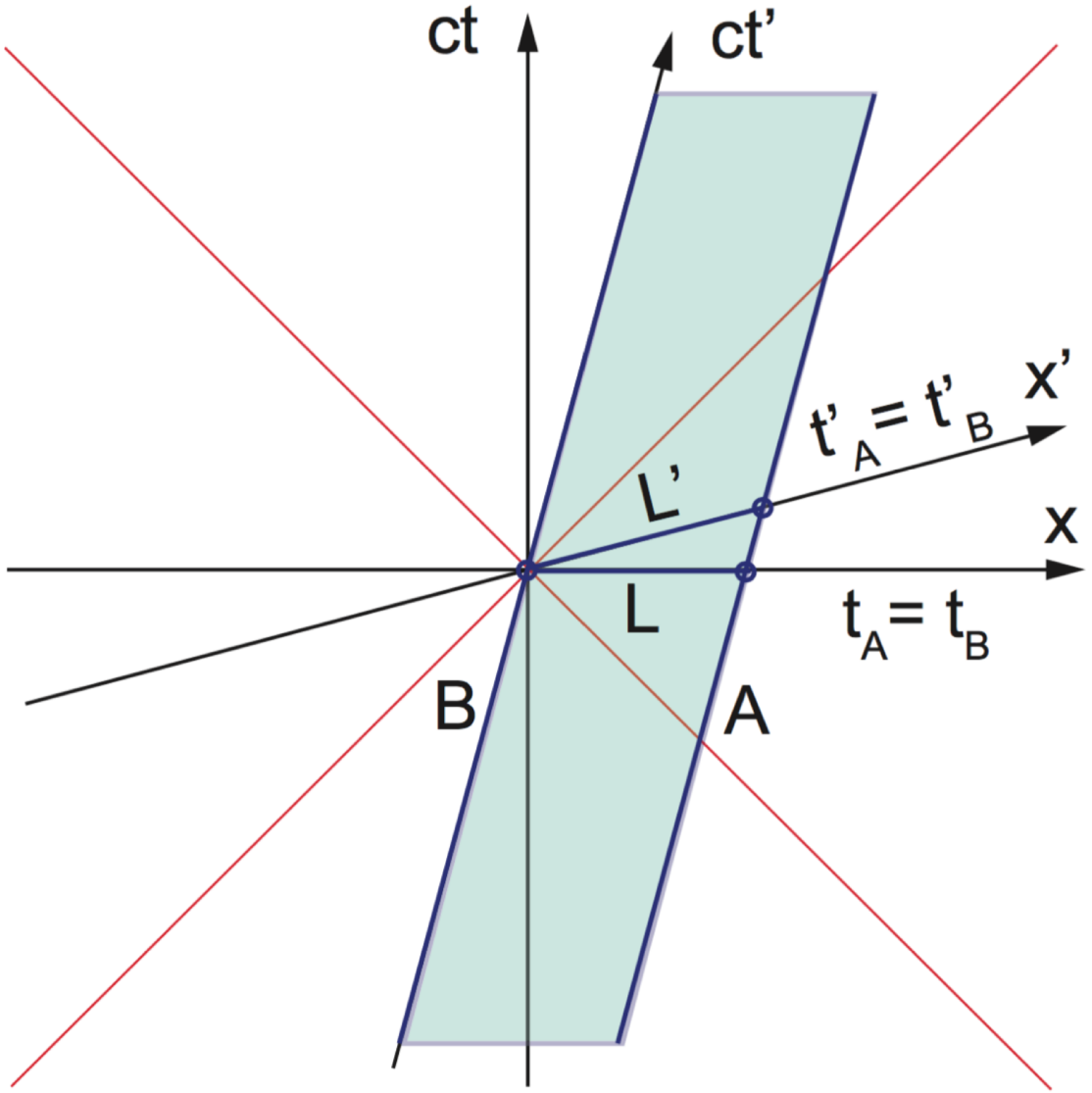
$$L^{\mu}{}_{\nu} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Adding translations we have the **Poincaré transformation** $x'^{\mu} = L^{\mu}{}_{\nu} x^{\nu} + a^{\mu}$

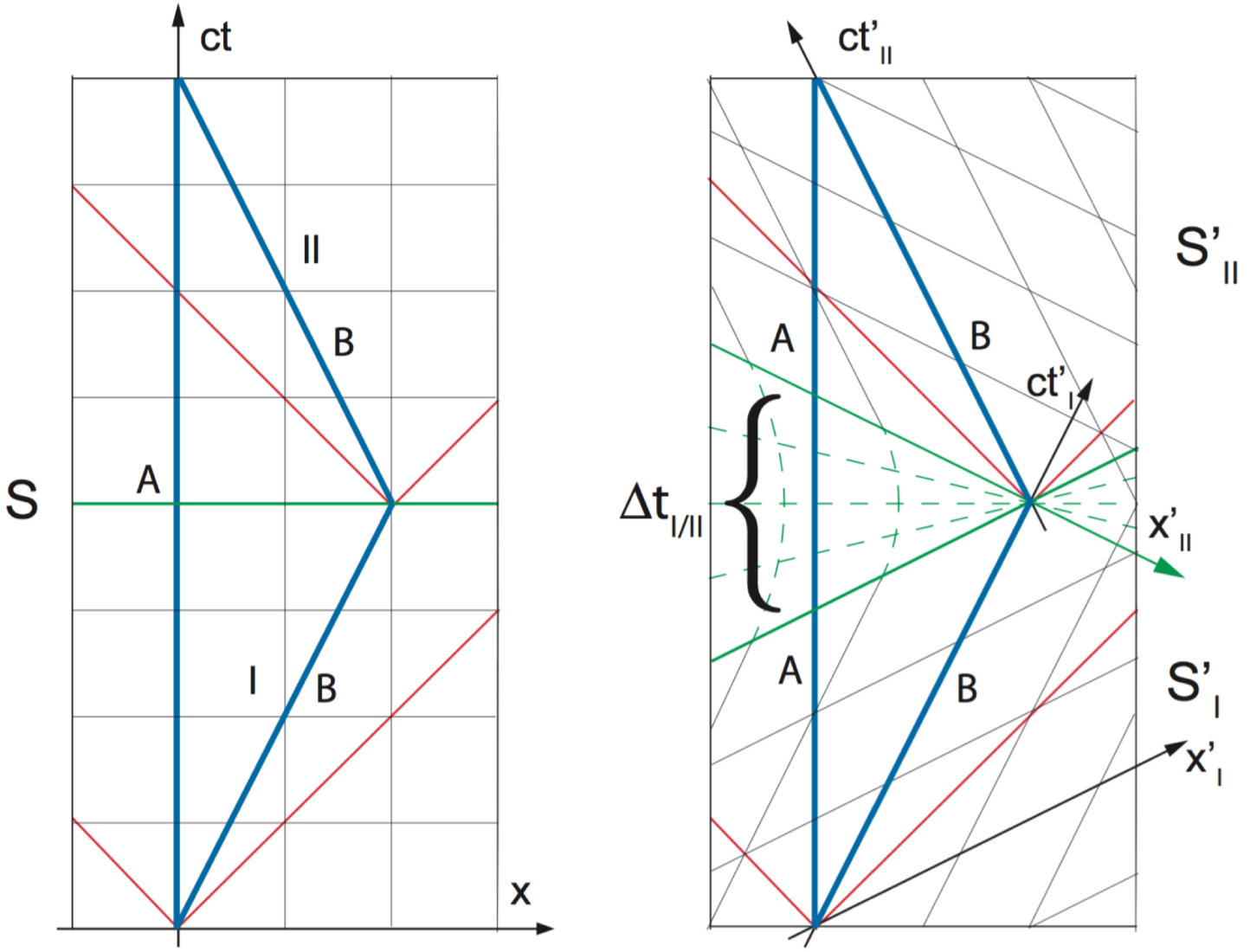
Plan for today

- Length contraction
  - The length of objects is different in different RFs!
- Time dilatation
  - Time moves differently in different RFs!
- Proper time
  - How to get a good definition of time even when accelerating.
- The twin paradox (*sigh*)
  - A completely bloody annoying useless example of nothing.

Length contraction



Twin paradox



Summary

- A body of length L_0 at rest in RF S' moving with velocity v w.r.t. RF S has length L in S given by

$$L = \frac{1}{\gamma} L_0 \leq L_0$$

A time interval τ in S' is the interval t in S

$$t = \gamma \tau \geq \tau$$

This is **length contraction** and **time dilation**.

- The **proper time** is given as

$$\tau_{AB} \equiv \int_{t_A}^{t_B} \sqrt{1 - \frac{v^2(t)}{c^2}}\, dt$$