UiO : Fysisk institutt

Det matematisk-naturvitenskapelige fakultet

Lecture 13

Recap

- A body of length L_0 at rest in RF S' moving with velocity v w.r.t. RF S has length L in S given by

  $$L = \frac{1}{\gamma} L_0 \leq L_0$$

  A time interval τ in S' is the interval t in S:

  $$t = \gamma \tau \geq \tau$$

  This is **length contraction** and **time dilation**.

- The **proper time** is given as

  $$\tau_{AB} \equiv \int_{t_A}^{t_B} \sqrt{1 - \frac{v^2(t)}{c^2}}\, dt$$

Today

- Return of the four-vectors
  - We repeat Einstein's summation convention.
  - The metric tensor.
  - Lower indices (finally).
  - The general four-vector. (Not necessarily space-time x^μ.)
- Lorentz transformations strike again
  - Now properly using four-vectors and the metric.

Summary

- We define the metric tensor g and inverse g^{-1} as

$$g = g^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

- This is used to raise and lower indices

$$A^{\mu} = g^{\mu\nu} A_{\nu}, \quad A_{\mu} = g_{\mu\nu} A^{\nu}$$

- Lorentz transformations are given by

$$A'^{\mu} = L^{\mu}{}_{\nu} A^{\nu}, \quad A'_{\mu} = L_{\mu}{}^{\nu} A_{\nu}$$

where L fulfils $g_{\mu\nu} L^{\mu}{}_{\rho} L^{\nu}{}_{\sigma} = g_{\rho\sigma}$