UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 14

This week

- **Monday:** Tensors and vector fields. (Sections 6.5-6.6)
- **Wednesday:** Relativistic kinematics with four-vectors. (Section 7.1)
- **Problem session:** Problem set 6, Minkowski diagrams for trains and devilish boosts.
- I also added a problem to the set of extra problems for Part I.

Recap

- We define the metric tensor g and inverse g^{-1} as

$$g = g^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

- Used to raise and lower four-vector indices

$$A^{\mu} = g^{\mu\nu} A_{\nu}, \quad A_{\mu} = g_{\mu\nu} A^{\nu}$$

- Lorentz transformations are given by

$$A'^{\mu} = L^{\mu}{}_{\nu} A^{\nu}, \quad A'_{\mu} = L_{\mu}{}^{\nu} A_{\nu}$$

where L fulfils $g_{\mu\nu} L^{\mu}{}_{\rho} L^{\nu}{}_{\sigma} = g_{\rho\sigma}$

Today

- Tensors
  - Four-vectors with (possibly) more indices.
  - Rank (number of indices).
  - Tensor product.
- Vector and tensor fields
- Differentiation for four-vectors
  - The d'Alembertian.

Summary

- We can generalize four-vectors to rank-n tensors with n indices, e.g. the rank-2 $F^{\mu\nu}$.
- We also have tensor fields, e.g. $\varphi(x)$, $A^\mu(x)$, $F^{\mu\nu}(x)$, that transform as relativistic tensors.
- We define a four-component derivative as

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}; \qquad \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right)$$

  that also transforms as a covariant four-vector.
- This can be used to define the d'Alembertian

$$\partial_\mu \partial^\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$$