Lecture 19

This week

- **Monday:** A covariant Lagrangian formulation. (Section 8.4)
- **Wednesday:** Starting Part III. Maxwell's equations. (Sections 9.1-9.5).
- **Problem session:** Problem set 8. More practice on relativistic kinematics.
- **Mid-term:** results should be out some time this week.

Recap

- Relativistic four-force K^μ is defined as

$$K^\mu = \frac{dp^\mu}{d\tau} = \gamma\left(\frac{1}{c}\vec{v}\cdot\vec{F}\,,\ \vec{F}\right)$$

where the relativistic force F is

$$\vec{F} = \frac{d\vec{p}}{dt}\,, \qquad \vec{p} = \gamma m \vec{v}$$

- We can write the Lorentz force (force from electromagnetic field) on covariant form as

$$K^\mu = eF^{\mu\nu}U_\nu \qquad \left(F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu\,,\ \ A^\mu = \left(\phi/c\,,\vec{A}\right)\right)$$

where $F^{\mu\nu}$ is the electromagnetic field strength.

Today

- A covariant Lagrangian
  - The Lagrangian in terms of Lorentz vectors only.
  - Lagrange's equation on covariant form.
  - Example with free particle.
  - Example with free particle.
  - Example with free particle.
  - Example with particle in e.m. field.

Mid-term evaluation

- Lectures:
  - More examples in class.
  - Lorentz vectors: challenging; too little time to practice use.
- Problem sessions:
  - More problems, more types of problems.
  - Too short problem sets? (Opinion is divided.)
  - More discussion of concepts in problem classes.
  - General positive attitude to oral test instead of handing in written problem sets.

Summary

- We can write a relativistic action as

$$S = \int_{\tau_1}^{\tau_2} L(x^\mu(\tau), U^\mu(\tau), \tau)\, d\tau$$

which is a Lorentz (transformation) invariant scalar using a Lorentz invariant Lagrangian L.

- Lagrange's equations are then

$$\frac{d}{d\tau}\frac{\partial L}{\partial U^\mu} - \frac{\partial L}{\partial x^\mu} = 0$$

where τ is proper time and U^μ is four-velocity.