UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 5

Today

- Conservation of energy
  - From the time invariance of the Lagrangian
  - Introducing, for the very first time: the Hamiltonian! (or maybe not)
- More general Lagrangians
  - Adding a total time derivative (useful trick)
  - Allowing for velocity dependence in potential

Recap

- With the Lagrange function

$$L(q,\dot{q},t) = K(q,\dot{q},t) - V(q,t)$$

we can find the equations of motion for the system from Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \qquad i = 1,\ldots,d$$

Recap

- Cyclic coordinates are coordinates q_i that do not appear in the Lagrangian
- The corresponding **conjugate momentum** p_i is conserved

$$p_i \equiv \frac{\partial L}{\partial \dot{q}_i}$$

- If Q is the parameter of a transformation leaving the Lagrangian invariant the conserved quantity K is

$$K = \sum_i \frac{\partial L}{\partial \dot{q}_i} Q_i$$

Summary

- The Hamiltonian H for a Lagrangian with no explicit time dependence is a constant of motion $$H = \sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L$$
- When constraints are time-independent the Hamiltonian is given by H = K+V.
- Total time derivative terms in the Lagrangian can be ignored.
- For velocity dependent potentials (forces)
$$F_j = \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_i}\right) - \frac{\partial U}{\partial q_i}$$