



UiO : **Fysisk institutt**

Det matematisk-naturvitenskapelige fakultet

Lecture 6



This week

- **Monday:** Lagrangian for a charged particle in an electromagnetic field. (Section 2.5)
- **Wednesday:** Hamilton's equations, Hamiltonian for charged particles in an electromagnetic field. (Sections 3.1 and 3.2)
- **Problem session:** Problem set 3 (main topic: even more e.o.m. from Lagrange's equation, cyclic coordinates and how to remove them)

Recap

- No explicit time dependence in the Lagrangian L

$$\frac{\partial L}{\partial t} = 0,$$

means that the Hamiltonian

$$H \equiv \sum_i \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i - L,$$

is a constant of motion/conserved quantity.

- When constraints are time- and velocity independent (holonomic) the Hamiltonian is given by $H = K+V$.

Recap

- Total time derivative terms in the Lagrangian do not change the e.o.m.
- For velocity dependent potentials (forces) the generalized force is written as

$$F_j = \frac{d}{dt} \left(\frac{\partial U}{\partial \dot{q}_i} \right) - \frac{\partial U}{\partial q_i}$$

leading to the Lagrange function $L = K - U$ in Lagrange's equation.

Plan for today

- Charged particle in an electromagnetic field.
(Section 2.5)
 - Electromagnetic potentials instead of fields (short preview of Section 9.5).
 - The force on a free charged particle in terms of a velocity dependent “potential” U .
 - The Lagrangian.
 - The conjugate momentum.
 - The Hamiltonian.
 - A symmetry of the Lagrangian – gauge invariance.

Summary

- The electromagnetic (e.m.) potentials give a potential energy
$$U = e\phi - e\vec{v} \cdot \vec{A}$$

- The conjugate momentum is

$$\vec{p} = m\vec{v} + e\vec{A}$$

and the Hamiltonian

$$H = \frac{1}{2m} (\vec{p} - e\vec{A})^2 + e\phi$$

- The e.o.m. are invariant under the gauge transformations

$$\phi \rightarrow \phi' = \phi - \frac{\partial \chi}{\partial t}, \quad \vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla} \chi$$