UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 8

This week

- **Monday:** Application of Hamilton's equations, phase space. (Sections 3.2, 3.3, and 3.5)
- **Wednesday:** Hamilton's principle and calculus of variations. (Section 3.6)
- **Problem session:** Problem set 4 (more Lagrangians).

Recap

- Expressing the Hamiltonian H in terms of the canonical position and momentum q_i and p_i the e.o.m can be written as Hamilton's equations

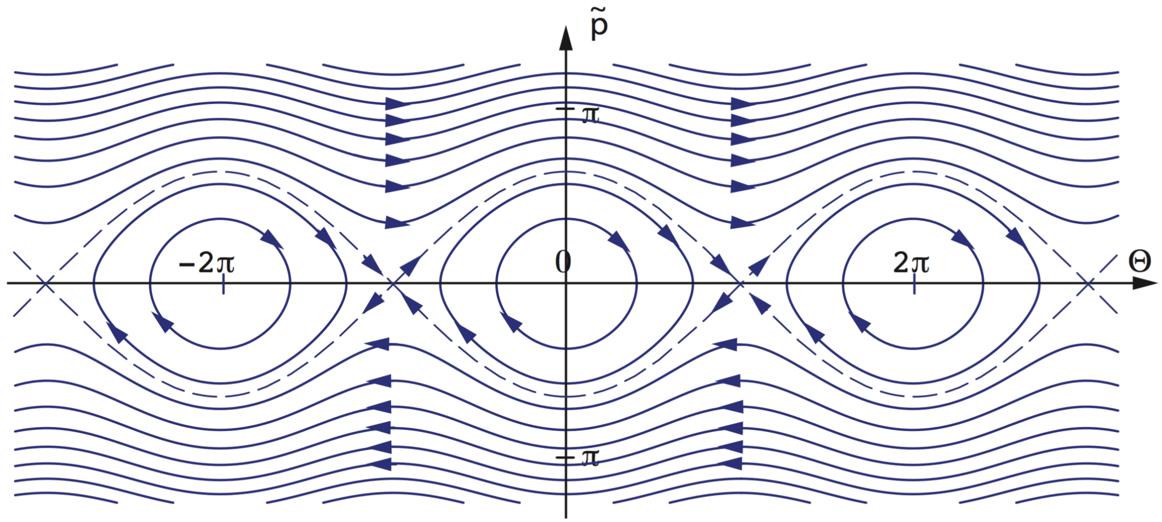
$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}, \qquad i=1,\ldots,d$$

- To simplify calculations it is useful to introduce the totally antisymmetric Levi-Civita symbol

$$\epsilon_{ijk} \equiv \begin{cases} 1 & \text{for } i \neq j \neq k \text{ and cyclic} \\ -1 & \text{for } i \neq j \neq k \text{ and not cyclic} \\ 0 & \text{otherwise} \end{cases}$$

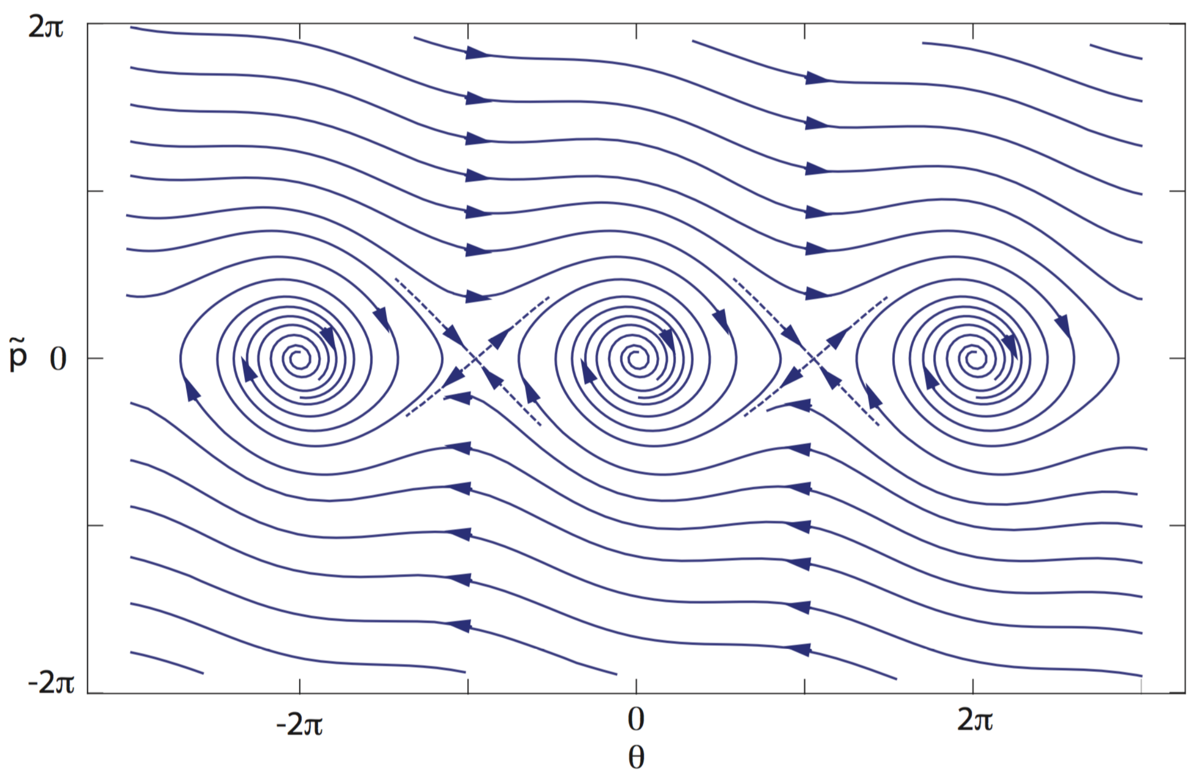
Plan for today

- Charged particle in a constant magnetic field Oh no! Not again! (Section 3.2)
  - Against all previous experience, we are actually going to solve the e.o.m. this time.
- Phase space. (Section 3.3)
  - A nice way to think about motion.
- ~~Phase space fluid. (Section 3.4)~~
- Non-Hamiltonian systems. (Section 3.5)
  - What to do without energy conservation (if time!)



Pendulum

$$\tilde{p} = \frac{p}{m\sqrt{gl^3}}$$



Damped pendulum

Summary

- Phase space is the 2d-dimensional space of generalized coordinates and their velocities (q, qdot), or the space of generalized coordinated and generalized momenta (q,p).
- An initial value point in phase space gives (almost always) a unique trajectory.
- Analysis of phase space is very useful for a qualitative understanding of a problem.
  - Can be used even for non-Hamiltonian systems.