UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

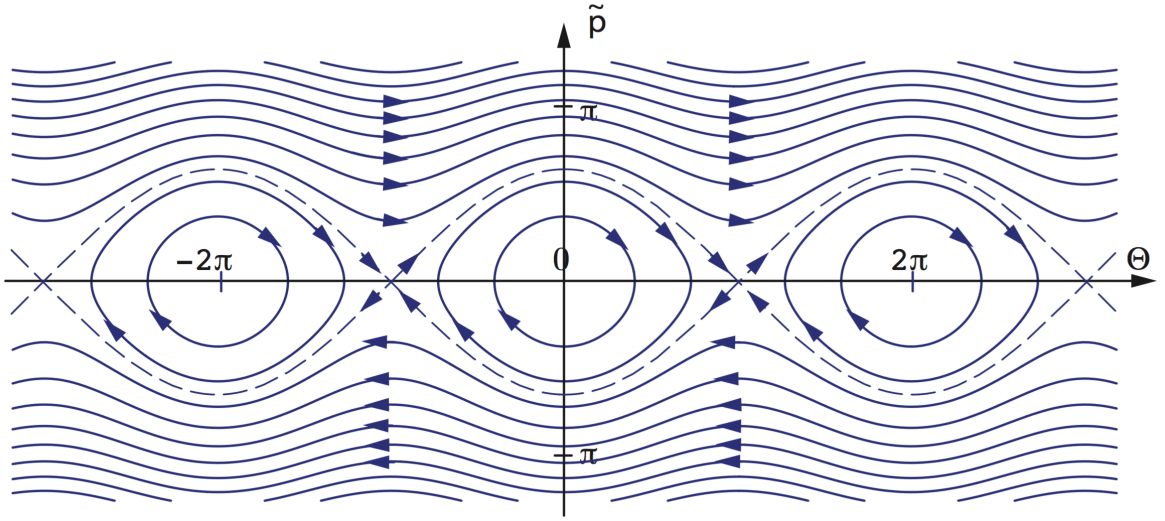
Lecture 9

Recap

- **Phase space** is the 2d-dimensional space of generalized coordinates and their velocities (q, qdot), or the space of generalized coordinated and generalized momenta (q,p).
- An initial value point in phase space gives (almost always) a unique trajectory.
  - Exception: unstable equilibria.
- Analysis of phase space is very useful for a qualitative understanding of a problem.

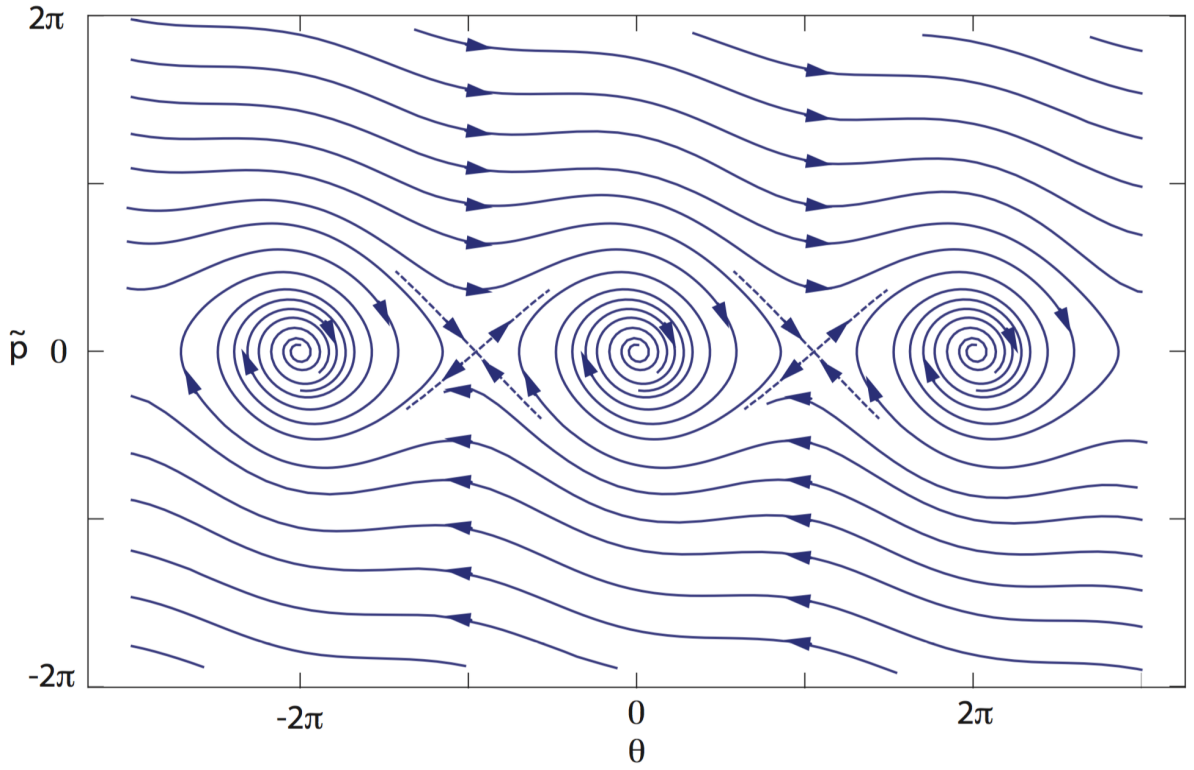
Today

- Non-Hamiltonian systems
  - When energy is not conserved
- Hamilton's principle
  - The action and the principle of least (no?) action.
  - Equivalence to Lagrange's equations.
  - Use of variational calculus outside of mechanics.
  - Poisson brackets (if time).



Pendulum

$$\tilde{p} = \frac{p}{m\sqrt{gl^3}}$$



Damped pendulum

Summary

- **Hamilton's principle** or **the principle of least action** says that the action

$$S[q(t)] = \int_{t_1}^{t_2} L(q(t), \dot{q}(t), t)\, dt$$

as a function of the path q(t) is unchanged for small variations

$$q(t) \rightarrow q(t) + \delta q \quad \text{with} \quad \delta q(t_1) = \delta q(t_2) = 0$$

around the trajectory that fulfils the e.o.m.

- This is equivalent to Lagrange's equations.