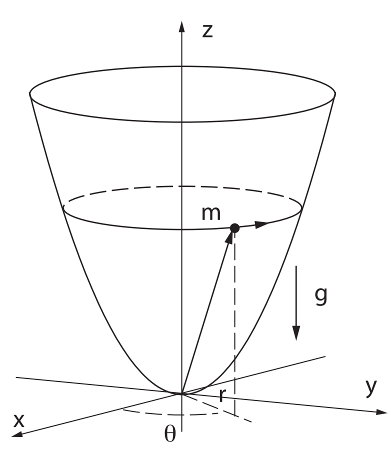
UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Tuesday June 3, 2008
Exam hours: 3 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Øgrim og Lian or Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120
Language: The solutions may be written in Norwegian or English depending on your own preference.

Make sure that your copy of this examination paper is complete before you begin.

PROBLEM 1
Particle on a constrained surface
A particle moves on a parabolic surface given by the equation $z = (\lambda/2)(x^2 + y^2)$ where z is the Cartesian coordinate in the vertical direction, x and y are orthogonal coordinates in the horizontal plane and λ is a constant. The particle has mass m and moves without friction on the surface under influence of gravitation. The gravitational acceleration g acts in the negative z-direction. The particle's position is given by the polar coordinates (r, θ) of the *projection* of the position vector into the x, y plane.



a) Show that the Lagrangian for this system is

$$L = \frac{1}{2}m[(1 + \lambda^2 r^2)\dot{r}^2 + r^2\dot{\theta}^2 - g\lambda r^2] \qquad (1)$$

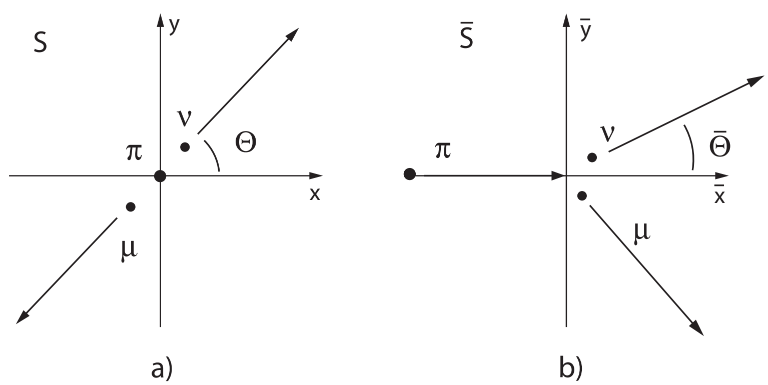
and find Lagrange's equations for the particle.

b) Use the fact that there is a cyclic coordinate to show that the equations can be reduced to a single equation in the radial variable r. What is the condition for the particle to move in a circle with radius $r = r_0$?

c) Assume that the path of the particle deviates little from the circular motion so that $r = r_0 + \rho$, where ρ is small. Show that under this condition the radial equation can be reduced to a harmonic oscillator equation for the small variable ρ and determine the corresponding frequency. Give a qualitative description of the motion of the particle.

PROBLEM 2
Particle decay

Pi-mesons (pions) are unstable elementary particles. We consider here a decay process of a charged pion π^+ into a muon μ^+ and a neutrino ν_μ. The masses of the particles are $m_\pi = 273 m_e$ and $m_\mu = 207 m_e$, with $m_e = 0.51 MeV/c^2$ as the electron (rest) mass. (The standard energy unit in particle physics, eV = electron volt is used. The speed of light is as usual represented by c.) The mass of the neutrino is so small that the particle can be regarded as massless.



In the figure the decay process is shown both in the rest frame S of the pion, and in the laboratory frame $\bar{S}$. In this frame the pion moves with the velocity $v = (4/5)c$ along the x axis. To distinguish the variables of the two reference frames S and $\bar{S}$ we mark the variables of the latter with a "bar", so that for example the angle of the neutrino relative to the x axis in S is θ and the corresponding angle in $\bar{S}$ is $\bar{\theta}$.

a) We study first the process in the rest frame S. Set up the equations for conservation of relativistic energy and momentum and use them to determine the energy and (the absolute value of) the momentum of the muon and of the neutrino in this reference system. (Use MeV as unit for energy and MeV/c as unit for momentum.)

b) Use the transformation formula for relativistic 4-momenta to determine the energy of the muon and of the neutrino in the lab frame $\bar{S}$.

c) In the rest frame S all directions for the neutrino momentum are equally probable. Show that this means that in the lab frame $\bar{S}$ the probability is 0.5 for finding the neutrino in a direction with angle $\bar{\theta} < 36.9^o$.

PROBLEM 3
Electric dipole radiation

An electron (with charge e and mass m) is moving with constant speed in a circle under the influence of a constant magnetic field $\mathbf{B}_0$. The magnetic field is directed along the z axis while the motion of the electron takes place in the x, y plane. We assume the motion of the electron to be non-relativistic.

Since the electron is accelerated it will radiate electromagnetic energy and thereby loose kinetic energy when no energy is added to the particle.

a) By use of Larmor's radiation formula, find an expression for the radiated energy per unit time expressed in terms of the radius r of the electron orbit and the cyclotron frequency $\omega = -eB_0/m$.

b) Show that the radius of the electron orbit is slowly reduced with an exponential form of the time dependence, $r = r_0\, e^{-\lambda t}$, and determine λ.

c) The electromagnetic fields produced by the moving charge are essentially electric dipole radiation fields. What is the electric dipole moment of the circulating electron? Give the expressions for the radiation fields $\mathbf{E}(z, t)$ and $\mathbf{B}(z, t)$ on the z axis far from the electron. Show that they correspond to a propagating wave, with direction away from the electron, and determine the form of polarization of the wave.

Expressions found in the formula collection of the course may be useful for this problem.