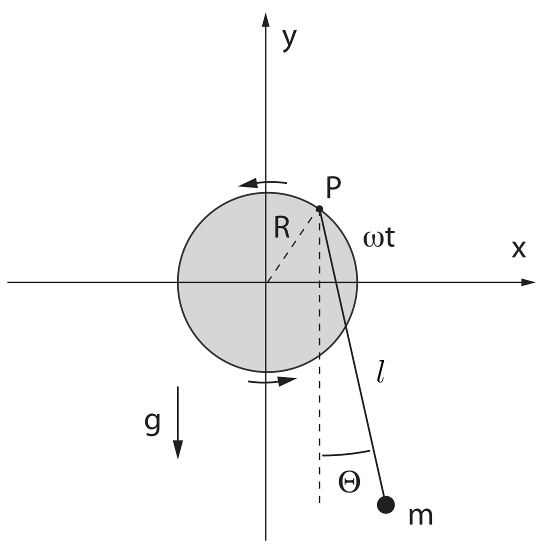
UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Tuesday June 2, 2009
Exam hours: 3 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Øgrim og Lian or Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120
Language: The solutions may be written in Norwegian or English depending on your own preference.

Make sure that your copy of this examination paper is complete before you begin.

PROBLEM 1
Pendulum attached to a rotating disk
A pendulum is attached to a circular disk of radius R, as illustrated in Fig. 1. The end of the pendulum rod is fixed at a point P on the circumference of disk. The disk is vertically oriented and it rotates with a constant angular velocity ω. The pendulum consists of a rigid rod of length l which we consider as massless and a pendulum bob of mass m. The pendulum oscillates freely about the point P under the influence of gravity.



a) Show that the Lagrangian for this system, when using as variable the angle θ of the pendulum rod relative to the vertical direction, has the form

$$L = m[\frac{1}{2}l^2\dot{\theta}^2 + lR\omega\sin(\theta - \omega t)\dot{\theta} + gl\cos\theta + \frac{1}{2}R^2\omega^2 - gR\sin\omega t] \tag{1}$$

b) Formulate Lagrange's equation for the system and write it as a differential equation for θ.

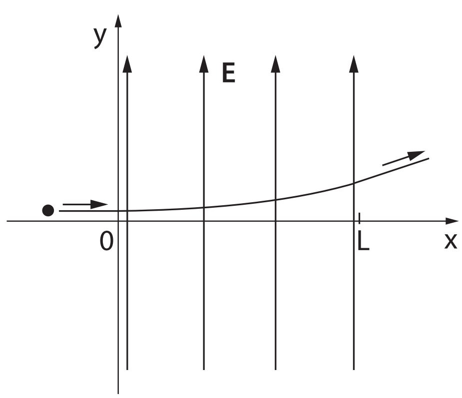
For $\omega = 0$ the equation reduces to a standard pendulum equation. Assume in the following ω to be non-vanishing, but sufficiently small so the ω-dependent contribution to the equation of motion can be viewed as a small periodic perturbation to the pendulum equation. In that case there are solutions corresponding to small oscillations, $|\theta| << 1$, which are modified by the perturbation.

c) Show that under assumption that $|\theta|$ and ω are sufficiently small the equation of motion for the pendulum can be approximated by the equation for a *driven* harmonic oscillator, subject to a periodic force. Show that it has a solution of the form $\theta(t) = \theta_o \cos\omega t$ and determine the amplitude θ_0 in terms of the parameters of the problem.

Based on this solution can you give a more precise meaning to the phrase "sufficiently small ω" as the condition for $\theta_o \cos\omega t$ to be a good approximation to a solution of the full equation of motion?

PROBLEM 2
Charged particle in a constant electric field

A particle with charge q and rest mass m moves with relativistic speed through a region $0 < x < L$ where a constant electric field $\mathbf{E}$ is directed along the y-axis, as indicated in the figure. The particle enters the field at $x = 0$ with momentum $\mathbf{p}_0$ in the direction orthogonal to the field. The relativistic energy at this point is denoted $\mathcal{E}_0$. (Note that we write the energy as $\mathcal{E}$ to avoid confusion with the electric field strength E.)



a) Use the equation of motion for a charged particle in an electric field to determine the time dependent momentum $\mathbf{p}(t)$ and relativistic energy $\mathcal{E}(t)$ (without the potential energy) of the particle inside in the electric field. What is the relativistic gamma factor $\gamma(t)$ expressed as a function of coordinate time t?

b) Find the velocity components $v_x(t)$ and $v_y(t)$ and explain the relativistic effect that the velocity in the x-direction decreases with time even if there is no force acting in this direction.

c) Show that the proper time $\Delta\tau$ spent by the particle on the transit through the region $0 < x < L$ is proportional to the length L, $\Delta\tau = \alpha L$, and determine α.

d) What is the transit time Δt through the region when measured in coordinate time?

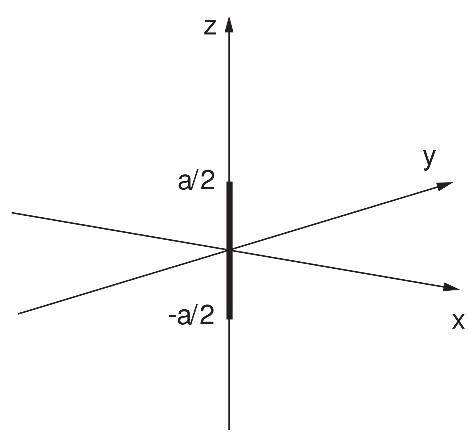
We remind about the integration formula $\int dx \frac{1}{\sqrt{1+x^2}} = \operatorname{arc\,sinh} x + C$.

PROBLEM 3
Radiation from a linear antenna

A so-called *half-wave center-fed* antenna is formed by a thin linear conductor of length a. It is oriented along the z-axis as shown in the figure. An alternating current is running in the antenna, of the form

$$I(z,t) = I_0 \cos\frac{\pi z}{a} \cos\omega t \ , \quad -a/2 < z < a/2 \tag{2}$$

In the following $\lambda(z,t)$ denotes the linear charge density of the antenna (charge per unit length). At time $t = 0$ the antenna is charge neutral, so that $\lambda(z,0) = 0$.



a) Show that the charge density and current satisfy the relation

$$\frac{\partial\lambda}{\partial t} + \frac{\partial I}{\partial z} = 0 \tag{3}$$

and find λ as a function of z and t.

b) Show that the electric dipole moment of the antenna has the form

$$\mathbf{p}(t) = p_0 \sin\omega t \ \mathbf{k} \tag{4}$$

with $\mathbf{k}$ as the unit vector along the z-axis, and determine the constant p_0.

c) Use the expressions for electric dipole radiation to determine the electric and magnetic fields in a point at a large distance r from the antenna on the x-axis. What is the type of polarization of the radiation from the antenna in this direction?