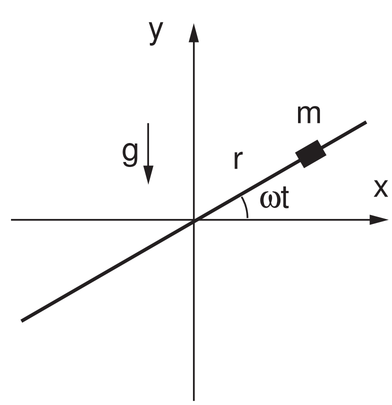
UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Tuesday June 4, 2013
Exam hours: 4 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Angell/Øgrim og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120
Language: The solutions may be written in Norwegian or English depending on your own preference. En kort ordliste finnes etter oppgavene.

Make sure that your copy of this examination paper is complete before answering .

PROBLEM 1
Constrained motion
A small body with mass m is constrained to move on straight line, which rotates with constant angular velocity in a vertical plane, here identified as the xy-plane. The y-axis is pointing in the vertical direction, as shown in the figure. The point of rotation, which lies on the line, is chosen as origin. At $t = 0$ the rotating line coincides with the x-axis. Friction is assumed to be negligible, and the only applied force acting on the body is the gravitational force mg. In the following the radial coordinate r is chosen as generalized coordinate, and the body is treated as pointlike.



a) Find the Lagrangian for this system, expressed in terms of r, $\dot{r}$ and t. Derive the corresponding Lagrange's equation.

b) Show that the equation has a particular solution of the form $r = k \sin \omega t$ and determine the constant k. Show that the solution corresponds to circular motion with constant speed, with the circle centered at the point $(x, y) = (0, k/2)$. What is the speed of the particle and the radius of the circle?

c) Find the general solution of the equation of motion.

PROBLEM 2
Hyperbolic space-time motion

A straight rod is moving along the x-axis of an inertial reference frame S. The two endpoints A and B follow hyperbolic space-time trajectories, described the following time dependent x-coordinates in S,

$$x_A = c\sqrt{t^2 + c^2/a^2}\,, \quad x_B = c\sqrt{t^2 + c^2/b^2} \tag{1}$$

c is the speed of light, and a and b are positive constants, with $b < a$.

a) A second inertial frame S' moves along the x-axis with velocity v relative to S. The coordinates of the two reference frames are chosen to coincide at the space-time point $x = t = 0$.

Show that the motion of A and B, when expressed in terms of the coordinates of S', has precisely the same form as in S,

$$x'_A = c\sqrt{t'^2 + c^2/a^2}\,, \quad x'_B = c\sqrt{t'^2 + c^2/b^2} \tag{2}$$

(To demonstrate this it may be convenient to rewrite the above relations in terms of the squared coordinates x^2 and t^2.)

b) At time $t = 0$ the frame S is an instantaneous rest frame of both A and B. Show this and find the distance between A and B measured in S at this moment. The same results are valid for the reference frame S' at time $t' = 0$.

Based on this we may conclude that for any point on the space-time trajectory of A, the instantaneous inertial rest frame of A is a rest frame also for B. Furthermore the distance between A and B, when measured in the instantaneous inertial rest frame, is constant. Explain these conclusions.

c) Use the above results to show that the proper accelerations of the A and B are constants, and give the values of these.

d) At a given instant $t = 0$ a light signal with frequency ν_0 is sent from A and is subsequently received at B. What is the velocity of B (measured in S) when the signal is received, and what is the frequency of the signal, measured at B? (To answer the last question it may be convenient to use the relation between frequency and four-momentum for a photon sent from A to B.)

PROBLEM 3
Radiation from a current loop

In a circular loop of radius a an oscillating current of the form $I = I_0 \cos \omega t$ is running. The current loop lies in the x, y-plane, with the center of the loop at the origin. The loop is at all times charge neutral. We assume $a\,\omega << c$.

a) Show that the magnetic dipole moment has the following time dependence, $\mathbf{m}(t) = m_0 \cos\omega t\, \mathbf{e}_z$, with m_0 as a constant and $\mathbf{e}_z$ as a unit vector in the z-direction. Find m_0 expressed in terms of a and I_0. Far from the current loop, the fields are dominated by the magnetic dipole radiation field. Explain why.

b) The magnetic dipole radiation fields have the general form

$$\mathbf{E}(\mathbf{r},t) = -\frac{\mu_0}{4\pi c r}\ddot{\mathbf{m}}_{ret} \times \mathbf{n}\,;, \quad \mathbf{B}(\mathbf{r},t) = -\frac{1}{c}\mathbf{E}(\mathbf{r},t) \times \mathbf{n} \tag{3}$$

with $\mathbf{m}_{ret} = \mathbf{m}(t - r/c)$, $\mathbf{n} = \mathbf{r}/r$, and $r >> a$. For the present case give the full expressions of the fields, as functions of r and t, and written in terms of the orthonormalized vectors $\{\mathbf{n} = \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi\}$ of the polar coordinate system. Do they form, as expected, waves that propagate in the radial direction away from the loop? Explain. Characterize the polarization of the waves.

c) Use the general expression for Poynting's vector $\mathbf{S}$ to find the radiated power per unit solid angle $\frac{dP}{d\Omega}$, expressed as a function of the polar angle θ (angle relative to the z-axis). Find the total radiated power, integrated over all directions and averaged over time.

d) A second conducting loop, identical to the first one, is placed at a large distance r from the first loop, with the center of the loop in the x, y-plane. It is used as a receiver, with the radiation from the first loop inducing a current in the second loop. Let $\mathbf{u}$ be a unit vector orthogonal to the plane of the second loop. In what direction should $\mathbf{u}$ be oriented for the second loop to receive the maximal signal?

ORDLISTE

engelsk	**norsk**
Lagrangian	Lagrangefunksjon
angular velocity	vinkelhastighet
instantaneous rest frame	momentant hvilesystem
proper acceleration	egenakselerasjon
current loop	strømsløyfe
radiated power	utstrålt effekt