UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Monday June 2, 2014
Exam hours: 4 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection/Formelsamling FYS 3120
Language: This paper is available also in Norwegian (Bokmål or Nynorsk).

Make sure that your copy of this examination paper is complete before answering.

PROBLEM 1
Charged particle motion in a potential

A small body with mass m and charge q is moving in the horizontal plane (x, y-plane), under influence of a harmonic oscillator potential, $V(r) = \frac{1}{2}m\omega_0^2 r^2$ and a constant magnetic field $\mathbf{B} = B\,\mathbf{k}$, which is directed perpendicular to the plane of the moving particle. The vector potential corresponding to $\mathbf{B}$ can be wrtten as $\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}$, with $\mathbf{r}$ as the postion vector of the particle.

a) With the polar coordinates of the plane (r, ϕ) used as as generalized coordinates, show that the Lagrangian takes the form

$$L = \frac{1}{2}m(\dot{r}^2 + r^2(\dot{\phi}^2 + \omega_B\dot{\phi} - \omega_0^2)) \tag{1}$$

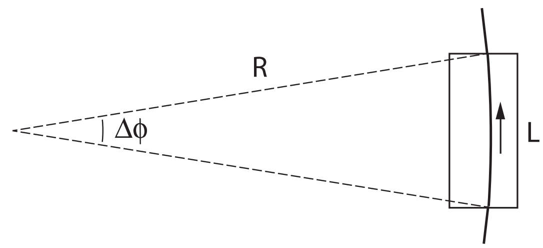
where we have introduced the cyclotron frequency, $\omega_B = qB/m$.

b) The polar angle ϕ is cyclic. Explain what that means and give the expression for the corresponding conserved quantity, which we lable l. What is the physical interpretation of the quantity? The form of the Lagrangian implies that there is a second constant of motion. Give the expression and physical interpretation of this quantity.

c) Establish Lagrange's equation for the variable r, and use the cyclic property of ϕ to express the equation in the variable r alone.

d) Show that the radial equation has solutions which describe circular motion, and give that radius and angular velocity of the motion as functions of the parameters of the problem. Show also that it has a solution where the particle performs oscillations about the origin, in a direction which rotates with with time, and find the oscillation and rotation frequencies. Give a qualitative description of the more general type of motion described by the equation.

PROBLEM 2
Protons in LHC
Protons in the accelerator ring LHC are bent into a near circular orbit by a large number of strong magnets. We consider in this problem the motion of a proton within one of the magnets. A strong magnetic field $\mathbf{B}$ perpendicular to the plan of the ring will bend the orbit with a bending radius R, as illustrated in the figure. We consider the magnetic field inside the magnet to be constant in strength.



For the accelerator we have the following information. The proton momentum is $p = 7\,\mathrm{TeV/c}$ (or $pc = 7 \cdot 10^{12}\,\mathrm{eV}$), the bending radius of the magnet is $R = 2804\mathrm{m}$, and the strength of the magnetic field is $B = 8.33\,\mathrm{T}$. The proton mass is $m = 938\,\mathrm{MeV/c^2}$, and the speed of light is $c = 3.0 \cdot 10^8\,\mathrm{m/s}$.

a) Show that we have the following relation between the strength of the magnetic field and the bending radius

$$eB = \frac{p}{R} \tag{2}$$

b) Find the relativistic gamma factor γ of the proton, and the acceleration a of the particle within the magnet, both determined in the laboratory frame, where the accelerator ring is at rest.

c) We consider the same situation in the instantaneous rest frame of the proton. What is the strength and orientation of the magnetic field $\mathbf{B}'$ and the electric field $\mathbf{E}'$ in this reference frame, and what is the proper acceleration a_0 of the proton?

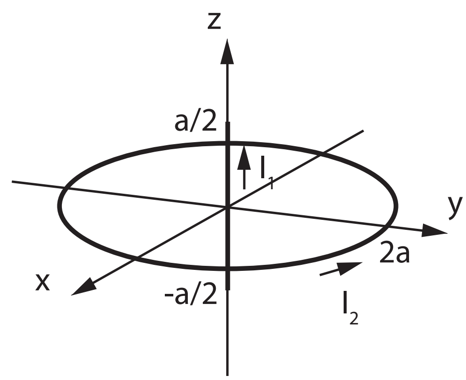
PROBLEM 3
Dipole radiation from an antenna
An antenna is composed of two parts, as shown in the figure. One part is a linear antenna along the z-axis, with end points $z = \pm a/2$. It caries the current

$$I_1 = I_0 \sin \omega t \cos \frac{\pi z}{a} \tag{3}$$

The other part is a circular antenna, which lies in the x,y-plane, and is centered at the origin of the coordinate system. It has radius $2a$ and carries the current

$$I_2 = I_0 \sin \omega t \tag{4}$$

The linear charge density $\lambda(z,t)$ of the linear antenna is determined by the continuity equation



for charge

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I_1}{\partial z} = 0 \tag{5}$$

while the circular one is all the time charge neutral.

We assume that the radiation from the antenna is dominated by electric and magnetic dipole radation.

a) Show that the time derivative of the electric dipole moment is given by

$$\dot{\mathbf{p}} = \frac{2}{\pi} a I_0 \sin \omega t \, \mathbf{k} \tag{6}$$

and the magnetic dipole moment is

$$\mathbf{m} = 4\pi a^2 I_0 \sin \omega t \, \mathbf{k} \tag{7}$$

As a reminder, the dipole contributions to the electric and magnetic fields, are in the radiation zone given by

$$\mathbf{E}(\mathbf{r}, t) = \frac{\mu_0}{4\pi r}((\ddot{\mathbf{p}} \times \mathbf{n}) \times \mathbf{n} - \frac{1}{c}\ddot{\mathbf{m}} \times \mathbf{n} + ...)_{ret}\,, \quad \mathbf{B}(\mathbf{r}, t) = -\frac{1}{c}\mathbf{E}(\mathbf{r}, t) \times \mathbf{n} \tag{8}$$

with $\mathbf{n} = \mathbf{r}/r$.

b) Assume the frequency ω is chosen so that the time average of the power of the electric and magnetic dipole radiation from the antenna are equal. Find for this case the radiated power per unit solid angle, $\frac{dP}{d\Omega}$, expressed as a function of the angle θ between the vector $\mathbf{n}$ and the z-axis.

c) What is in this case the polarization of the radiation? If the frequency ω changes so that the time average of the power of the electric and magnetic dipole radiation are no longer equal, how would that influence the polarization?

d) Assume that the antenna described above acts as an emitter. A second antenna, with circular form, like one of the parts of the emitter antenna, is connected to a receiver, which is placed in the radiation zone. How should the plane of this antenna be oriented to receive the maximal signal from the emitter?