FYS3140. First obligatory set of problems V2011

Problem 1
Obtain the Laurent series around z=0 for the function

$$f(z)=\frac{z}{(z-1)(2-z)}$$

that is convergent for:

a) |z|<1
b) 1<|z|<2
c) |z|>2

Hint: Write $f(z)$ as a sum of two fractions.

Problem 2
Use Cauchys integral theorem, Cauchys integral formula, or the integral expression for the derivative to determine the value of the following integrals, all around the circle |z|=2:

a) $\oint \frac{\cos z}{z} dz$

b) $\oint \frac{e^z}{z-1} dz$

c) $\oint \frac{2z^2+3z-1}{z-1+i} dz$

d) $\oint \frac{e^z}{(z-1)^2} dz$

e) $\oint \frac{\sin z}{z^4} dz$ (answer: $-\frac{\pi i}{3}$)

Problem 3
Use the residue theorem to calculate the following integrals:

a) $\oint_C \frac{dz}{z^2+4}$, C: circle $|z-2i|=1$

b) $\oint_C \frac{\cosh \pi z}{z(z^2+1)} dz$, C: circle $|z|=2$ (answer: $4\pi i$)

c) $\oint_C z e^{\frac{1}{z}} dz$, C: circle $|z|=2$ (answer: πi)

Problem 4
Use the residue theorem to show that

$$\int_0^{2\pi} \frac{d\theta}{1+\sin^2\theta} = \pi\sqrt{2}$$

Problem 5
Determine the principal value of the integral

$$\int_{-\infty}^{\infty} \frac{\cos \pi x}{1-4x^2} dx \,.$$

Problem 6
Obtain the general solution of the differential equation

$$x^2 y' + 3xy = 1 \;.$$

Due Monday February 28th at 14.00. To be turned in at the department office. Write your name on your paper, not your candidate number.