Week 10 – Plan

- Highlights of last week
- Fourier series
- (Fourier transforms)

Problem set 8, due Mon 11/4 at 14:30.

Week 10 – Highlights of last week (1)

Series expansion methods – homogeneous DEs

- DEs with certain singularities [$b(x)$ and $c(x)$ below are analytic at $x = 0$] can be solved by a generalised power series approach, the Fröbenius method:

$$y'' + \frac{b(x)}{x} y' + \frac{c(x)}{x^2} y = 0.$$

- Solution of the form $y(x) = x^s \sum_{n=0}^{\infty} a_n x^n$
- The number s may be real or complex and is determined by the indicial equation [b_0 and c_0 are zeroth order of the Taylor expansions of $b(x)$ and $c(x)$) respectively]:

$$s(s-1) + b_0 s + c_0 = 0$$

- If the indicial equation gives two distinct solutions with $s_1 - s_2 = \mathit{integer}$: The smaller value often gives the full solution so try that one first.

Week 10 – Highlights of last week (2)

Fourier series

- Fourier series are series expansions of periodic functions, in the form (for period 2π)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

- Coefficients are determined using the orthogonality properties of sin and cos,

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$