Week 11 – Plan

- Highlights of last week
- Fourier transforms cont.
- Dirac delta function
- Laplace transformation

Problem set 9

Week 11 – Highlights of last week (1)

Fourier series

- Dirichlet conditions: Fourier series converges if $f(x)$ has a finite number of extrema, and a finite number of (finite) discontinuities in its basic interval, i.e. is bounded. At discontinuities the Fourier series converges to the mid point.
- Complex Fourier series:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

$$c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx} dx$$

- Other intervals, length $2L$:

$$x \to \frac{\pi x}{L}, \qquad \frac{1}{2\pi} \int_{-\pi}^{\pi} \to \frac{1}{2L} \int_{-L}^{L}$$

Week 11 – Highlights of last week (2)

Fourier series

- The Fourier series of an even function is a cosine series. The Fourier series of an odd function is a sine series. For a given function (given for $[0, L]$) one may choose to define an even or odd extension to the interval $[-L, L]$.

Week 11 – Highlights of last week (3)

Fourier transforms

- Continuum ($L \to \infty$) analog of Fourier series. Represents non-periodic functions

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{ikx} dk$$

- Prefactors are a convention (their product must be $1/2\pi$). Some books define with opposite signs in the exponentials.
- According to the *Fourier integral theorem* the inverse transformation is valid if $f(x)$ satisfies the Dirichlet conditions on any finite interval, and $\int_{-\infty}^{\infty} |f(x)| dx$ is finite. At discontinuities in $f(x)$, transforming to $F(k)$ and back gives the value at the midpoint of the jump.