Week 14 – Plan

- Highlights of last week
- Variational calculus cont.
- Partial differential equations: Separation of variables (1D wave equation)

Problem set 12, due May 9th.

Week 14 – Highlights of last week (1)

Tensors

- Inertia tensor I: $L_i = I_{ij}\omega_j$, derived from $\vec{L} = m\vec{r} \times (\omega \times \vec{r})$
- The nabla operator ∇ transforms as a vector.

Week 14 – Highlights of last week (2)

Tensors

- Levi Civita tensor ϵ_{ijk} equals ± 1 if ijk is an even (odd) permutation of 123, and is zero if two or three indices are identical.
- Useful sum rule:

$$\epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}.$$

- Very convenient tool in the context of vector identities (cross product, curl) and computing 3x3 determinants – takes care of the signs.

Week 14 – Highlights of last week (3)

Variational calculus

- Task: Given an integral of the type

$$I = \int_{x_1}^{x_2} F(x, y, y')dx, \quad y' = \frac{dy}{dx},$$

find $y(x)$ [or $x(y)$] between x_1 and x_2 such that I is stationary.

- Solution: Look at varied curves around the stationary one to derive *Euler-Lagrange (EL) equations*

$$\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0$$

- If $\partial F/\partial y = 0$, EL simplify to a first integral

$$\frac{\partial F}{\partial y'} = constant$$

- If $\partial F/\partial x = 0$, the same type of simplification is obtained by changing integration variable from x to y.

Week 14 – Highlights of last week (4)

Variational calculus

- Important examples of variational principles in physics I: Hamilton's principle (stationary action):

$$S = \int L(x, \dot{x}, t) dt, \quad L = T - V$$

- Important examples of variational principles in physics II: Fermat's principle:

$$P = \int n \, ds$$

where P is the "optical path", n the local index of refraction.