# Week 3 – Plan

Lectures:

- Highlights of last week
- Cauchy theorem
- Cauchy's integral formula. Examples
- Taylor series of analytic functions
- Laurent series (series representation near a singularity)
- (Zeros and singularities/poles)

Problem session: Problem set 2. Hand in by Monday 8th at 14:30.

# Week 3 – Highlights of last week (1)

**Analyticity:** $f(z) = u(x, y) + iv(x, y)$

- A function is analytic in a region of the complex plane if it has a (*unique*) derivative at every point of that region.
- Existence and uniqueness of the derivative is a very strong demand. From it one can derive the *Cauchy-Riemann equations* as a criterion for analyticity:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}; \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

- A point where $f$ is analytic is called regular point. A point where $f$ is not analytic is called singular point or singularity.
- If a function is analytic, i.e. has first derivatives, in a region, it possesses *derivatives of all orders* in that region.
- If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a region, then $u$ and $v$ are *harmonic*, i.e. solutions of the 2D Laplace equation.

# Week 3 – Highlights of last week (2)

**Integrals of complex functions**

- Integrals in the complex plane are along curves/contours. Can be defined via generalised Riemann sums.
- The first examples we did were solved by explicit parametrization of the contour.
- Upper bound estimate:

$$\left| \int_{\Gamma} f(z) dz \right| \leq ML$$

- Here $M$ is the maximum value of $f(z)$ on $\Gamma$, and $L$ is the length of $\Gamma$