Week 7 – Plan

- Highlights of last week
- Euler-Cauchy-equation (homogeneous, but non-constant coefficients)
- Non-homogeneous DEs: Background theory
- Constant coefficients on the LHS: Method of undetermined coefficients
- More general methods: Factorization, variation of parameters

Problem set 6. Hand in by Thursday 10/3 at 14:30 **NB!!**.

Week 7 – Highlights of last week (1)

Residue theory

- *Principal value:*
  Let Γ denote a semicircle of radius R in the upper half-plane and assume that $\left|\int_\Gamma f(z)dz\right| \to 0$ as $R \to \infty$. Then the principal value of an integral is given by:

$$PV \int_{-\infty}^{\infty} f(x)dx = 2\pi i \sum_k (Res(f; z_k)) + \pi i \sum_j Res(f; z_j)$$

- where z_k are singularities in the upper half-plane, and z_j are **simple** poles on the real axis.

Week 7 – Highlights of last week (2)

Ordinary differential equations (DEs)

- Linear first order equations, $F(x, y, y') = 0$: Solved by the method of integrating factors
- Second order DEs: $y'' + P(x)y' + Q(x)y = R(x)$.
- Homogeneous: $R(x) = 0$. The general solution is a linear combination of two linearly independent solutions, $c_1 y_1(x) + c_2 y_2(x)$. The solution is uniquely fixed if $y(x_0)$ and $y'(x_0)$ are specified.
- If one solution $y_1(x)$ of the homogeneous DE is known, a second, linearly independent one, can be found by variation of constants, $y_2(x) = c(x)y_1(x)$. This ansatz will lead to a *first* order DE for $c'(x)$, to be solved by integrating factors.

Week 7 – Highlights of last week (3)

Ordinary differential equations (DEs) – homogeneous case

- Homogeneous DEs with constant coefficients: General solution of the form $e^{\lambda x}$. Get a quadratic characteristic equation for λ. Roots can be real or complex. In the case of double roots, variation of constants gives $y(x) = (A + Bx)e^{\lambda x}$.