Week 8 – Plan

- Highlights of last week
- Greens functions
- Series expansion methods: Power series – Legendre's equation
- Series expansion methods: Frobenius method.

Home exam available Thursday (approx 4pm). Deadline Fri 18th 14:00

Week 1 – Highlights of last week (1)

Inhomogeneous differential equations (DEs)

- General solution: $y(x) = y_h(x) + y_p(x)$ where y_h is the full solution of the *homogeneous* equation, and y_p is any (particular) solution of the inhomogeneous equation, without arbitrary constants.
- Method of undetermined coefficients: If the LHS of the DE has constant coefficients, and the RHS $R(x)$ is a (combination of) 'simple' function(s) whose derivatives resemble the functions themselves, one can 'guess' a particular solution on the same form as $R(x)$, with unknown constant to be determined by inserting into the DE.

Week 8 – Highlights of last week (2)

Inhomogeneous differential equations (DEs)

- Find y_p from *factorisation*: If $u(x)$ is a solution of the homogeneous equation, then the ansatz $y_p(x) = u(x)v(x)$ will give a *first* order equation for $v'(x)$, which can thus be solved using integrating factors, and $v = \int v' dx$ gives v.
- Variation of parameters: If the full homogeneous solution $y_h(x) = c_1 y_1(x) + c_2 y_2(x)$ is known, the ansatz $y_p(x) = f_1(x)y_1(x) + f_2(x)y_2(x)$ leads to the explicit solution for y_p:

$$y_p(x) = -y_1 \int \frac{y_2 R}{W} dx + y_2 \int \frac{y_1 R}{W} dx$$

where $W = y_1 y_2' - y_2 y_1'$ is the Wronskian.