

Introduction to numerical projects

Here follows a brief recipe and recommendation on how to write a report for each project.

- Give a short description of the nature of the problem and the eventual numerical methods you have used.
- Describe the algorithm you have used and/or developed. Here you may find it convenient to use pseudocoding. In many cases you can describe the algorithm in the program itself.
- Include the source code of your program. Comment your program properly.
- If possible, try to find analytic solutions, or known limits in order to test your program when developing the code.
- Include your results either in figure form or in a table. Remember to label your results. All tables and figures should have relevant captions and labels on the axes.
- Try to evaluate the reliability and numerical stability/precision of your results. If possible, include a qualitative and/or quantitative discussion of the numerical stability, eventual loss of precision etc.
- Try to give an interpretation of your results in your answers to the problems.
- Critique: if possible include your comments and reflections about the exercise, whether you felt you learnt something, ideas for improvements and other thoughts you've made when solving the exercise. We wish to keep this course at the interactive level and your comments can help us improve it.
- Try to establish a practice where you log your work at the computerlab. You may find such a logbook very handy at later stages in your work, especially when you don't properly remember what a previous test version of your program did. Here you could also record the time spent on solving the exercise, various algorithms you may have tested or other topics which you feel worthy of mentioning.

Format for electronic delivery of report and programs

The preferred format for the report is a PDF file. You can also use DOC or postscript formats. As programming language we prefer that you choose between C/C++ and Fortran90/95. You could also use Java or Python as programming languages. Matlab/Maple/Mathematica/IDL are not allowed as programming languages for the handins, but you can use them to check your results where possible. The following prescription should be followed when preparing the report:

- Use Classfronter to hand in your projects, log in at blyant.uio.no and choose 'fellesrom fys3150 og fys4150'. Thereafter you will see an icon to the left with 'hand in' or 'innlevering'. Click on that icon and go to the given project. There you can load up the files within the deadline.
- Upload **only** the report file and the source code file(s) you have developed. The report file should include all of your discussions and a list of the codes you have developed. Do not include library files which are available at the course homepage, unless you have made specific changes to them.
- Comments from us on your projects, approval or not, corrections to be made etc can be found under your Classfronter domain and are only visible to you and the teachers of the course.

Finally, we do prefer that you work two and two together. Optimal working groups consist of 2-3 students. You can then hand in a common report.

Project 6, Transient flow between moving plates, deadline december 5 12pm (midnight)

For this project you will have to build your own program. The project is more numerically directed, with tests of algorithms for solving partial differential equations using finite difference schemes.

The physical problem can be that of the temperature gradient in a rod of length $L = 1$ or that of channel flow between two flat and infinite plates at $x = 0$ and $x = 1$, where the fluid is initially at rest and the plate $x = 1$ is given a sudden initial movement. We are looking at a one-dimensional problem

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, t > 0, x \in [0, L] \quad (1)$$

or

$$u_{xx} = u_t, \quad (2)$$

with initial conditions, i.e., the conditions at $t = 0$,

$$u(x, 0) = 0 \quad 0 < x < L \quad (3)$$

with $L = 1$ the length of the x -region of interest. The boundary conditions are

$$u(0, t) = 0 \quad t > 0, \quad (4)$$

and

$$u(L, t) = 1 \quad t > 0. \quad (5)$$

The function $u(x, t)$ can be the temperature gradient of a the rod or represent the fluid velocity in a direction parallel to the plates, that is normal to the x -axis. In the latter case, for small t , only the part of the fluid close to the moving plate is set in significant motion, resulting in a thin boundary layer at $x = 1$. As time increases, the velocity approaches a linear variation with x . In this case, which can be derived from the incompressible Navier-Stokes, the above equations constitute a model for studying friction between moving surfaces separated by a thin fluid film.

In this project we want to study the numerical stability of three methods for partial differential equations (PDEs). These methods are

1. The explicit forward Euler algorithm with discretized versions of time given by a forward formula and a centered difference in space resulting in

$$u_t \approx \frac{u(x, t + \Delta t) - u(x, t)}{\Delta t} = \frac{u(x_i, t_j + \Delta t) - u(x_i, t_j)}{\Delta t} \quad (6)$$

and

$$u_{xx} \approx \frac{u(x + \Delta x, t) - 2u(x, t) + u(x - \Delta x, t)}{\Delta x^2}, \quad (7)$$

or

$$u_{xx} \approx \frac{u(x_i + \Delta x, t_j) - 2u(x_i, t_j) + u(x_i - \Delta x, t_j)}{\Delta x^2}. \quad (8)$$

2. The implicit Backward Euler with

$$u_t \approx \frac{u(x, t) - u(x, t - \Delta t)}{\Delta t} = \frac{u(x_i, t_j) - u(x_i, t_j - \Delta t)}{\Delta t} \quad (9)$$

and

$$u_{xx} \approx \frac{u(x + \Delta x, t) - 2u(x, t) + u(x - \Delta x, t)}{\Delta x^2}, \quad (10)$$

or

$$u_{xx} \approx \frac{u(x_i + \Delta x, t_j) - 2u(x_i, t_j) + u(x_i - \Delta x, t_j)}{\Delta x^2}, \quad (11)$$

3. Finally we use the implicit Crank-Nicolson scheme with a time-centered scheme at $(x, t + \Delta t/2)$

$$u_t \approx \frac{u(x, t + \Delta t) - u(x, t)}{\Delta t} = \frac{u(x_i, t_j + \Delta t) - u(x_i, t_j)}{\Delta t}. \quad (12)$$

The corresponding spatial second-order derivative reads

$$u_{xx} \approx \frac{1}{2} \left(\frac{u(x_i + \Delta x, t_j) - 2u(x_i, t_j) + u(x_i - \Delta x, t_j)}{\Delta x^2} + \frac{u(x_i + \Delta x, t_j + \Delta t) - 2u(x_i, t_j + \Delta t) + u(x_i - \Delta x, t_j + \Delta t)}{\Delta x^2} \right). \quad (13)$$

Note well that we are using a time-centered scheme with $t + \Delta t/2$ as center.

- a) Write down the algorithms for these three methods and the equations you need to implement. For the implicit schemes show that the equations lead to a tridiagonal matrix system for the new values.
- b) Find the truncation errors of these three schemes and investigate their stability properties. Find also the analytic solution to the continuous problem.
- c) Implement the three algorithms in the same code and perform tests of the solution for these three approaches for $\Delta x = 1/10$, $h = 1/100$ using Δt as dictated by the stability limit of the explicit scheme. Study the solutions at two time points t_1 and t_2 where $u(x, t_1)$ is smooth but still significantly curved and $u(x, t_2)$ is almost linear, close to the stationary state.
- d) Compare the solutions at t_1 and t_2 with the analytic result for the continuous problem. Which of the schemes would you classify as the best?