

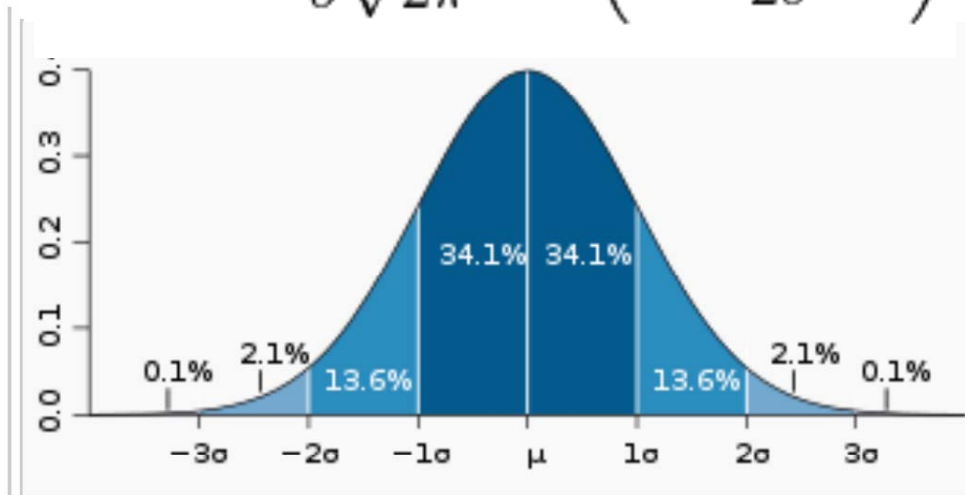
# Statistiske egenskaper ved målesystemer

Stoff fra

- Kompendiet
- Fraden 2.20
- Labøvelse 1

# Normalfordelingen

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$



Dark blue is less than one standard deviation from the mean. For the normal distribution, this accounts for about 68% of the set (dark blue), while two standard deviations from the mean (medium and dark blue) account for about 95%, and three standard deviations (light, medium, and dark blue) account for about 99.7%.

# Estimat av parametrene

Måler  $y$   $N$  ganger:

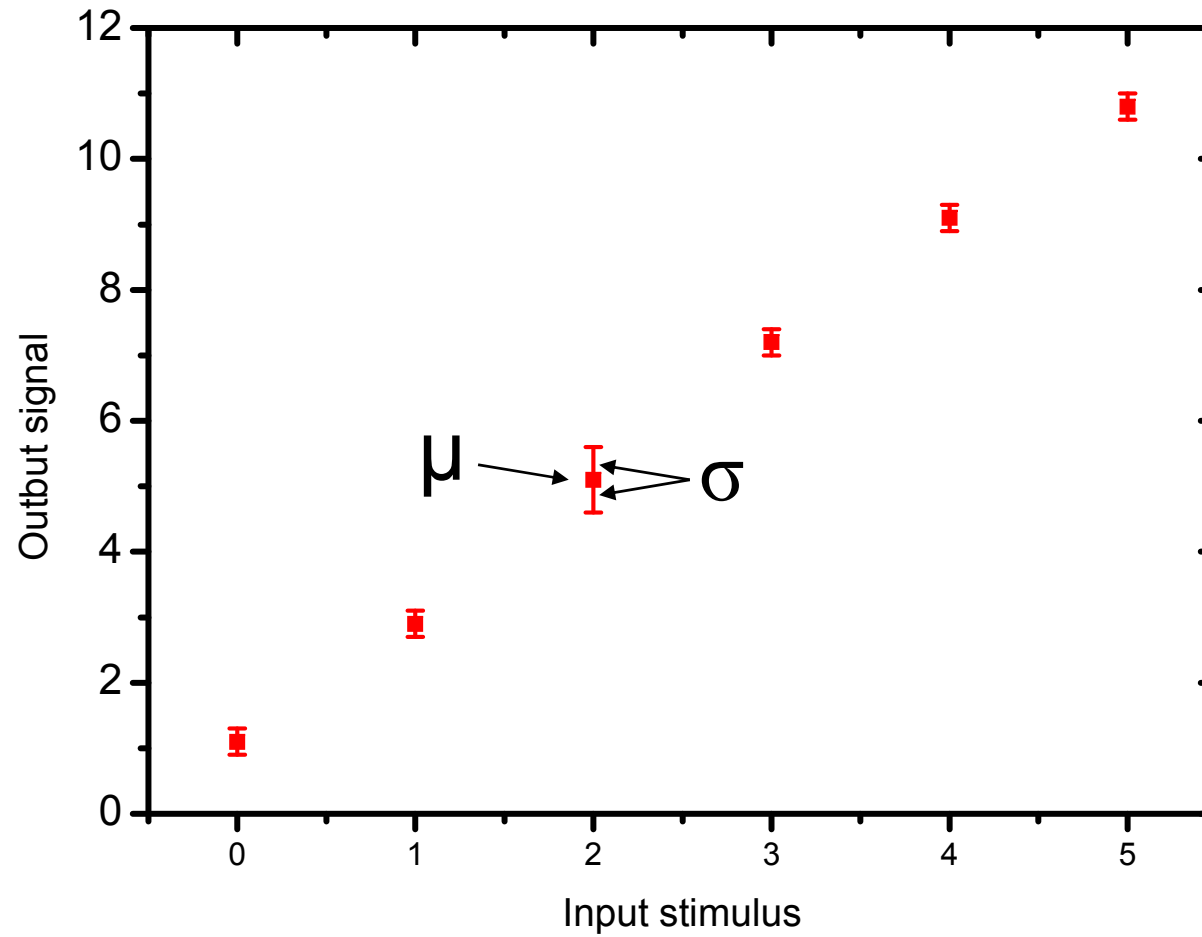
Middelverdi:

$$\mu = \frac{1}{N} \sum_{i=1}^N y_i = \langle y \rangle$$

Standard avvik:

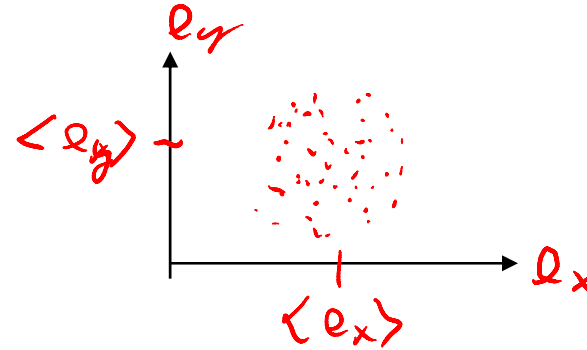
$$\begin{aligned} \sigma &= \sqrt{\frac{1}{N-1} \sum_{i=1}^N (y_i - \mu)^2} \\ &= \sqrt{\langle (y - \langle y \rangle)^2 \rangle} \end{aligned}$$

# Fremstilling av målinger (for varierende input)

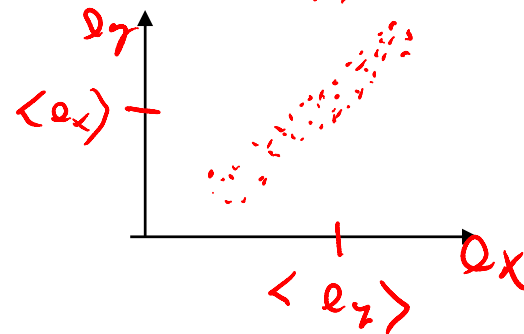


# Ko-varians

Ikke korrelerte størrelser

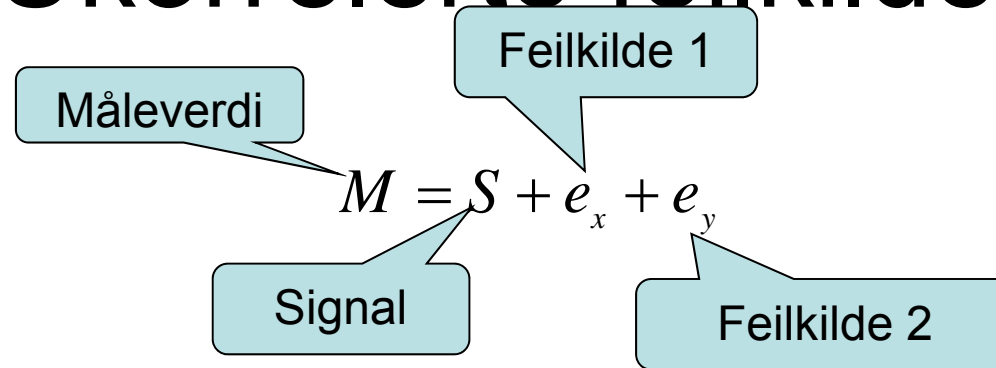


Korrelerte størrelser



$$\text{Cov}(e_x, e_y) = \langle (e_x - \langle e_x \rangle)(e_y - \langle e_y \rangle) \rangle$$

# Ukorrelerte feilkilder



Konstant signal  $\langle (S - \langle S \rangle)^2 \rangle = 0$   
Tilfeldig feil  $\langle e_x \rangle = \langle e_y \rangle = 0$

Ser på variansen av målesignalet

$$\begin{aligned} \langle (M - \langle M \rangle)^2 \rangle &= \langle ((S + e_x + e_y) - \langle (S + e_x + e_y) \rangle)^2 \rangle = \\ \langle ((\langle S \rangle + e_x + e_y) - (\langle S \rangle + \langle e_x \rangle + \langle e_y \rangle))^2 \rangle &= \langle (e_x + e_y)^2 \rangle = \\ \langle e_x^2 \rangle + 2 \underbrace{\langle e_x e_y \rangle}_{= \text{cov} = 0} + \langle e_y^2 \rangle \end{aligned}$$

# Usikkerhets budsjett

Table 2.2. Uncertainty Budget for Thermistor Thermometer

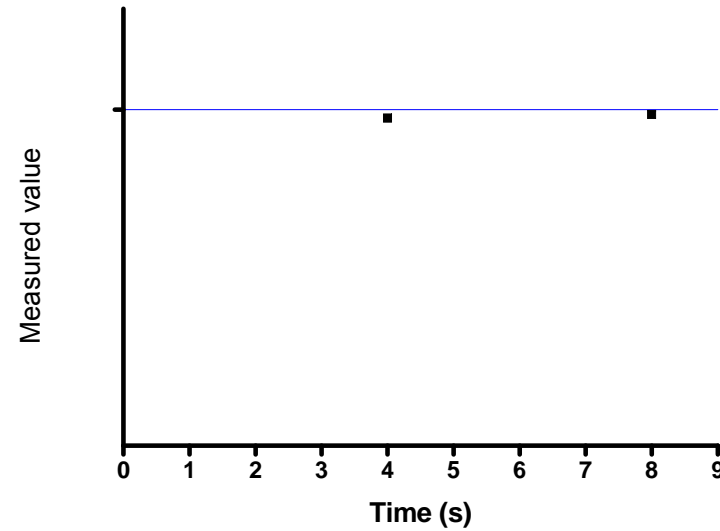
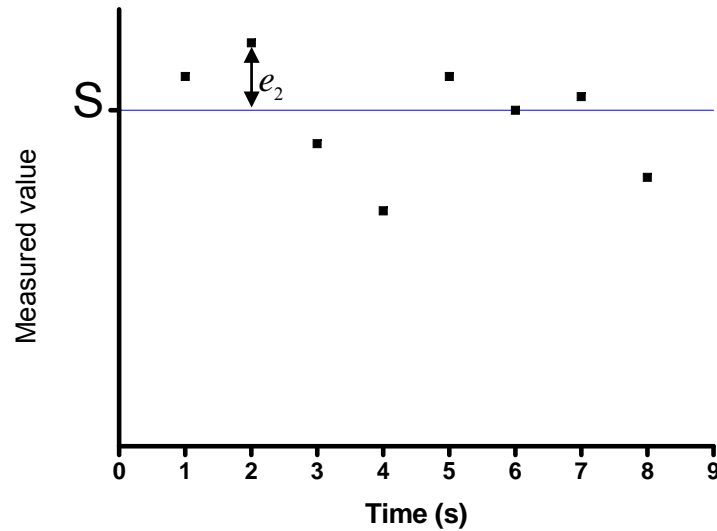
Source of Uncertainty	Standard uncertainty (°C)	Type
Calibration of sensor	0.03	B
Measured errors		
Repeated observations	0.02	A
Sensor noise	0.01	A
Amplifier noise	0.005	A
Sensor aging	0.025	B
Thermal loss through connecting wires	0.015	A
Dynamic error due to sensor's inertia	0.005	B
Temperature instability of object of measurement	0.04	A
Transmitted noise	0.01	A
Misfit of transfer function	0.02	B
Ambient drifts		
Voltage reference	0.01	A
Bridge resistors	0.01	A
Dielectric absorption in A/D capacitor	0.005	B
Digital resolution	0.01	A
<b>Combined standard uncertainty</b>	<b>0.068</b>	

$$u_c = \sqrt{u_1^2 + u_2^2 + \dots + u_i^2 + \dots + u_n^2}$$

(2.28)

A: Those evaluated by statistical methods  
B: Those evaluated by other means.

# Midling



$$M_i = S + e_i$$
$$\langle (M - \langle M \rangle)^2 \rangle = \langle e^2 \rangle$$

$$M = \frac{1}{4} \sum_{i=1}^4 M_i = \frac{1}{4} \sum_{i=1}^4 S + e_i$$

$$\langle (M - \langle M \rangle)^2 \rangle = ?$$



# Regner ut

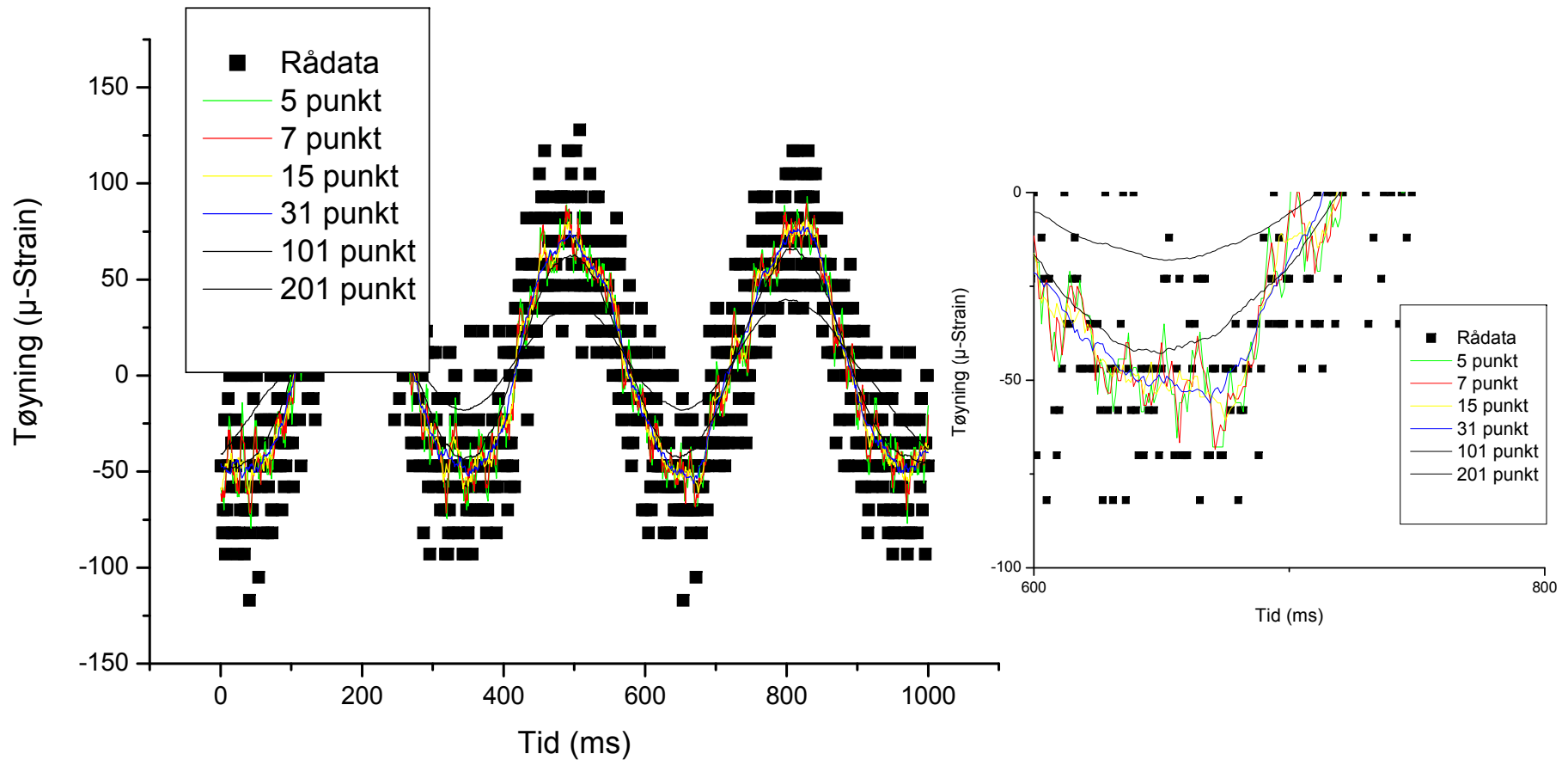
$$\begin{aligned}\langle (M - \langle M \rangle)^2 \rangle &= \left\langle \left( \left( \frac{1}{4} \sum_{i=1}^4 S + e_i \right) - \left\langle \left( \frac{1}{4} \sum_{i=1}^4 S + e_i \right) \right\rangle \right)^2 \right\rangle = \\ &= \frac{1}{16} \left\langle \left( (4\langle S \rangle + e_1 + e_2 + e_3 + e_4) - (4\langle S \rangle + \langle e_1 \rangle + \langle e_2 \rangle + \langle e_3 \rangle + \langle e_4 \rangle) \right)^2 \right\rangle = \frac{1}{16} \langle (e_1 + e_2 + e_3 + e_4)^2 \rangle \\ &= \frac{1}{16} \left( \underbrace{\langle e_1^2 \rangle + \langle e_2^2 \rangle + \langle e_3^2 \rangle + \langle e_4^2 \rangle}_{=\langle e^2 \rangle} + \underbrace{2\langle e_1 e_2 \rangle + \dots + 2\langle e_3 e_4 \rangle}_{=\text{cov}=0} \right) = \underline{\underline{\frac{1}{4} \langle e^2 \rangle}}\end{aligned}$$

Midling over 4 punkter reduserer **standard avviket** med en faktor 2

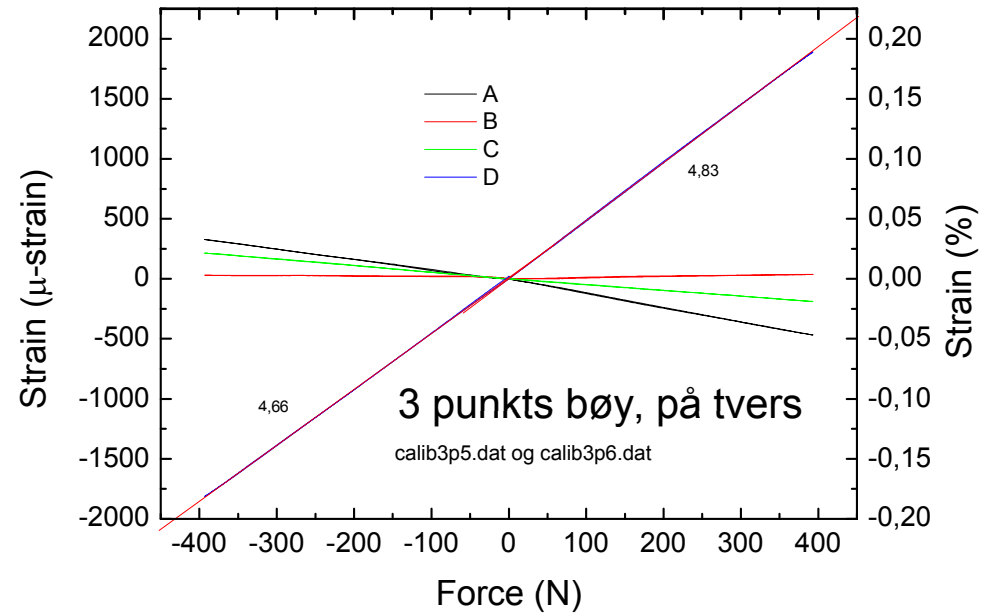
Lett å se at midling over n punkter reduserer **standard avviket** med en faktor  $\sqrt{n}$

# Glating - adjacent average

$$y(i) = \frac{1}{2n + 1} \sum_{n'=-n}^n y(i + n')$$



# Kalibrering av ishockeykølle



# Minste kvadraters metode

- Definerer avvik mellom modell og måling som

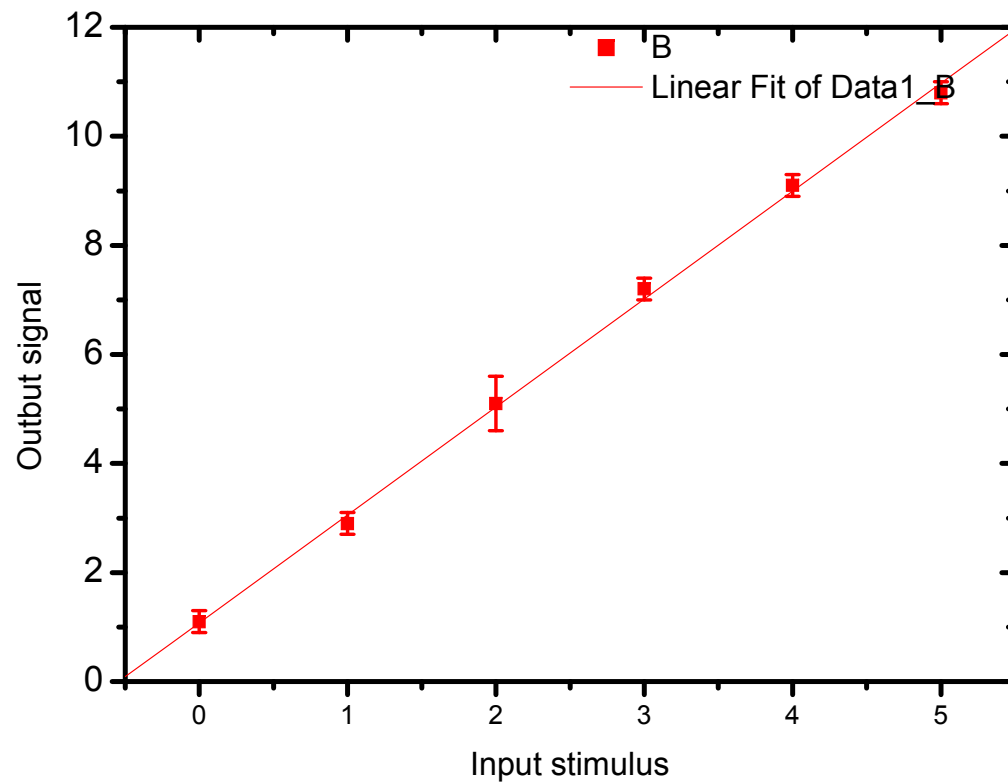
$$\chi^2 = \sum_{i=1}^N \left( \frac{y_i - y(x_i, a_1, \dots, a_M)}{\sigma_i} \right)^2$$

- Minimerer avviket med hensyn på parametrene i modellen ( $a_1 \dots a_n$ )
- Lineær tilpasning hvis  $y$  er lineært avhengig av  $a$

$$y(x) = a_1 + a_2x + a_3x^2 + \dots + a_Mx^{M-1}$$

=> Kan utlede formler for parametrene

# Lineær regresisjon med rett linje

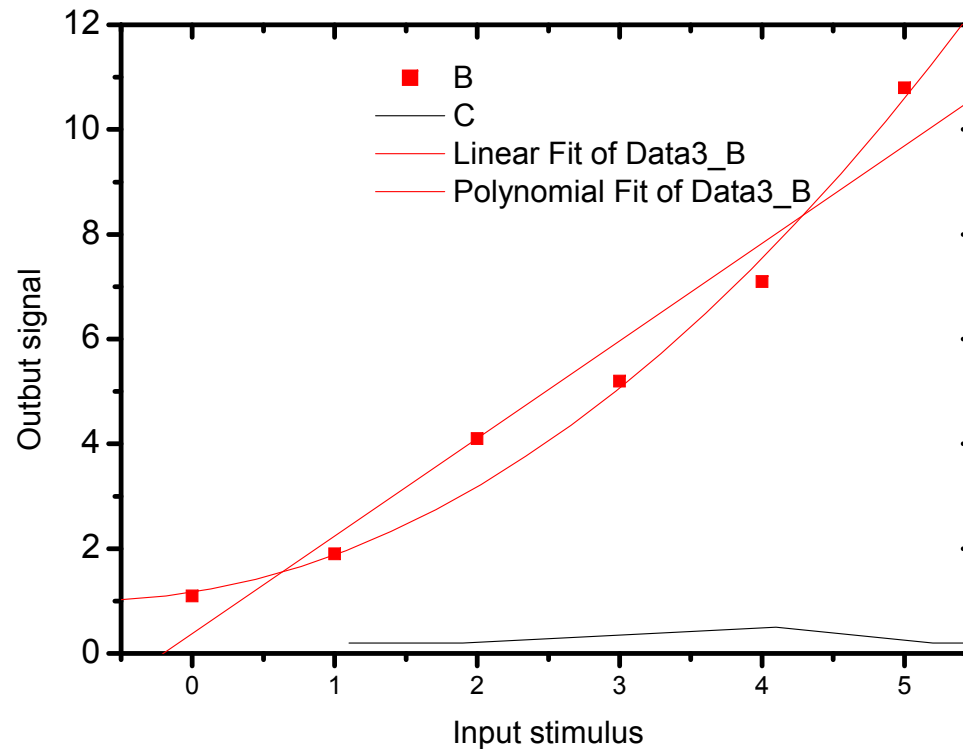


$$Y = A + B * X$$

Weight given by Data1\_C error bars.

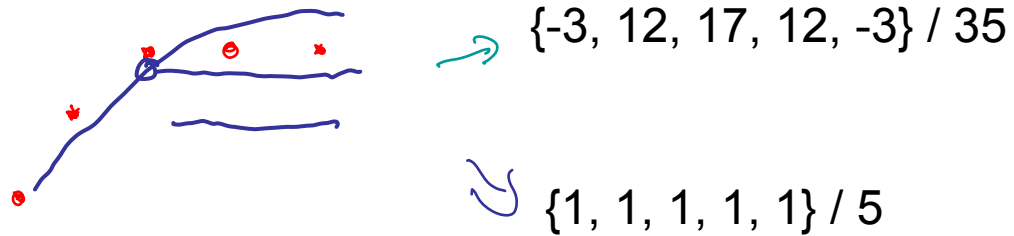
Parameter	Value	Error
A	1,07745	0,15231
B	1,97871	0,04815
R	SD	N
0,99925	0,79517	6

# Lineær regresjon med første og andre orden polynom



Forventet chi-kvadrat: Antall målepunkter minus antall frihetsgrader

# Savitzky-Golay



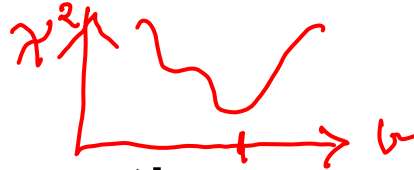
- Beste tilpasning til et polynom rundt punktet man ser på

# Ulineær kurvetilpasning

- Ulineære funksjoner  $y = a e^{-\left(\frac{x-b}{c}\right)^2}$

=>ingen formler for parametrene.

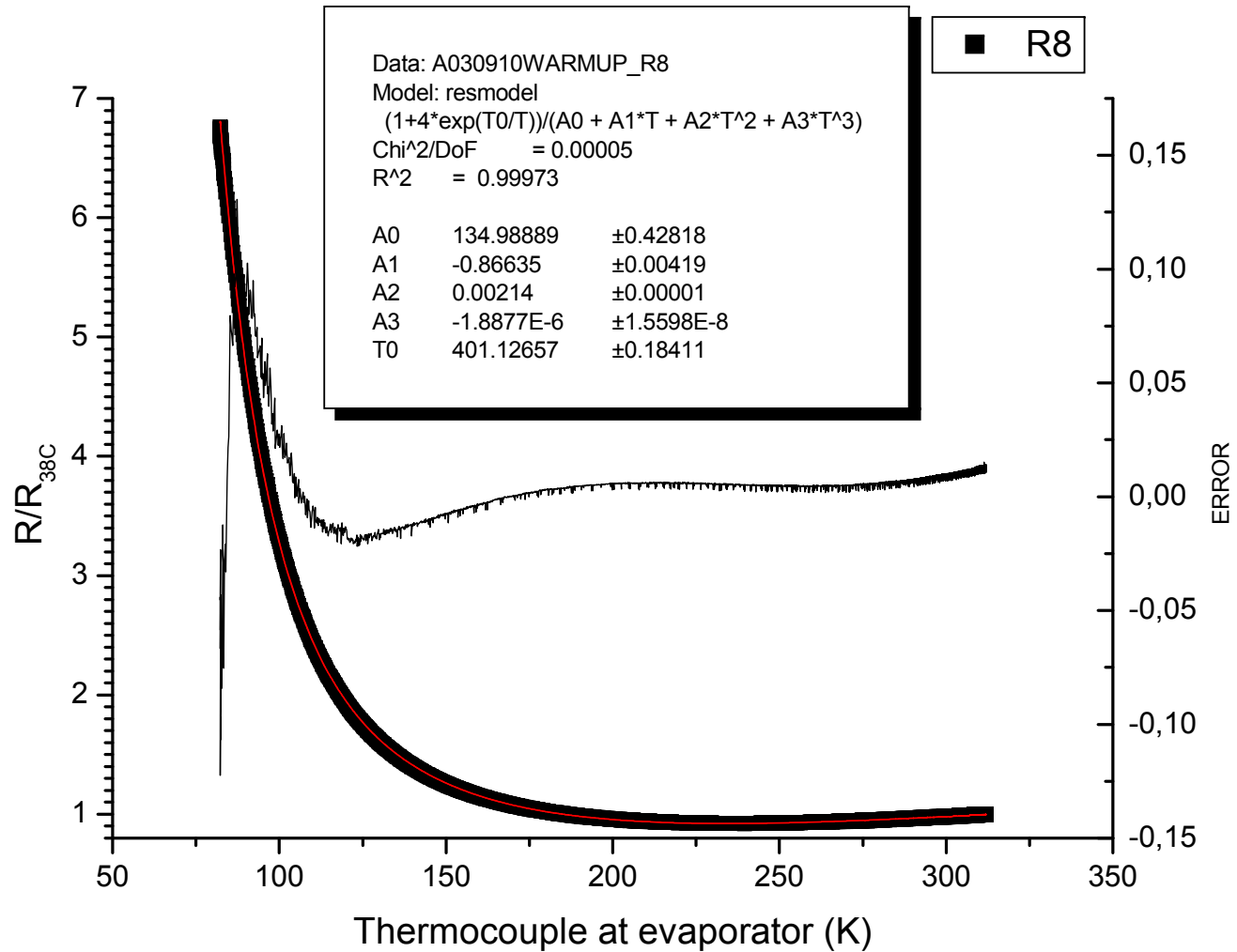
- Chi-kvadrat regnes ut numerisk og minimeres med hensyn på parametrene



- Ofte en iterativ prosess som krever gjetning og overvåking av feil



# Ulineær tilpasning og feil



# Ikke normalfordelte avvik

