Andre ordens system og vibrasjoner

- Hvordan måle
- Hvordan sette opp en modell
  - Sidespor – vibrasjoner
- Transferfunksjon
  - Elektrisk
  - Mekanisk
- Resonerende sensorer

- Section 3.14: Dynamic Models (Fraden)
- Section 8: Velocity and accelleration (Fraden)
- Paynter: Chapter 15

Transferfunksjon og impedans

I mekanikk er

- Transferfunksjonen forholdet mellom posisjon og kraft
- Impedans forholdet mellom hastighet og kraft

I elektronikk er (ofte)

- Transferfunksjonen forholdet mellom inngang og utgang
- Impedans forholdet mellom spenningen over kretsen og strømmen gjennom den

Fraden bruker transferfunksjon om den statiske karakteristikken til en sensor

For et svingesystem (andre ordens system) trenger vi

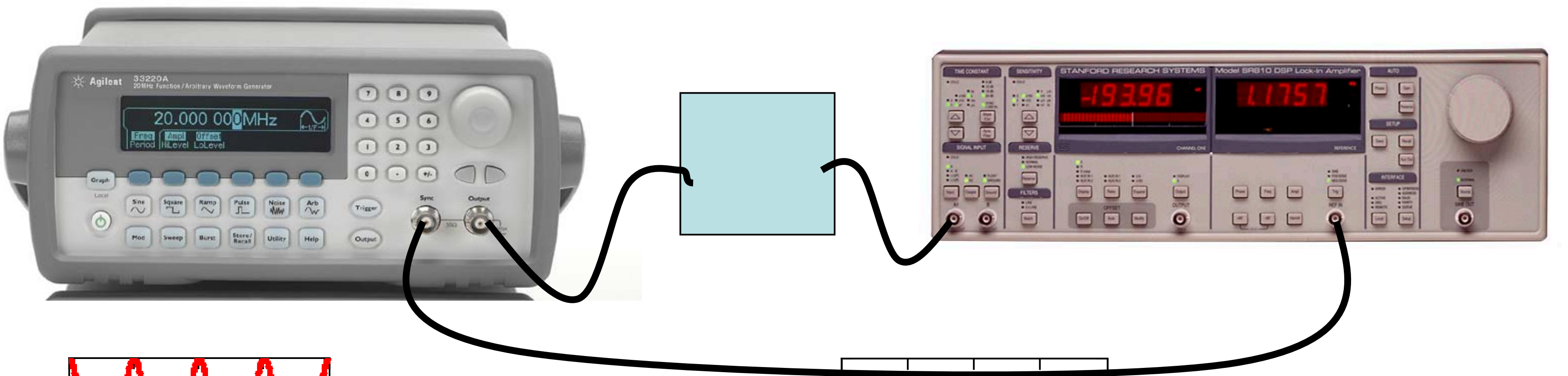
- To steder å lagre energi
  - Kondensator (elektrostatisk energi) og spole (magnetisk energi)
  - Masse i bevegelse (kinetisk energi) og fjær (potensiell energi)
- I tillegg har vi vanligvis et sted der energien blir gjort om til varme
  - Motstand
  - Demper

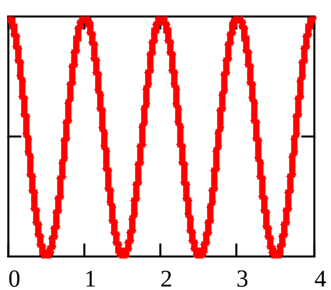
For å analysere et (svinge) system kan vi bruke

Signalgiver som gir et pådrag med bestemt
- (fase)
- frekvens
- amplitude

Analysator som kan finne
- fase
- amplitude
- (frekvens)

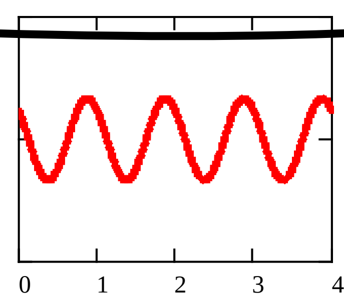
Av utgangssignalet





$$A_{inn} e^{j\omega t}$$

$$A_{inn} \cos(\omega t)$$



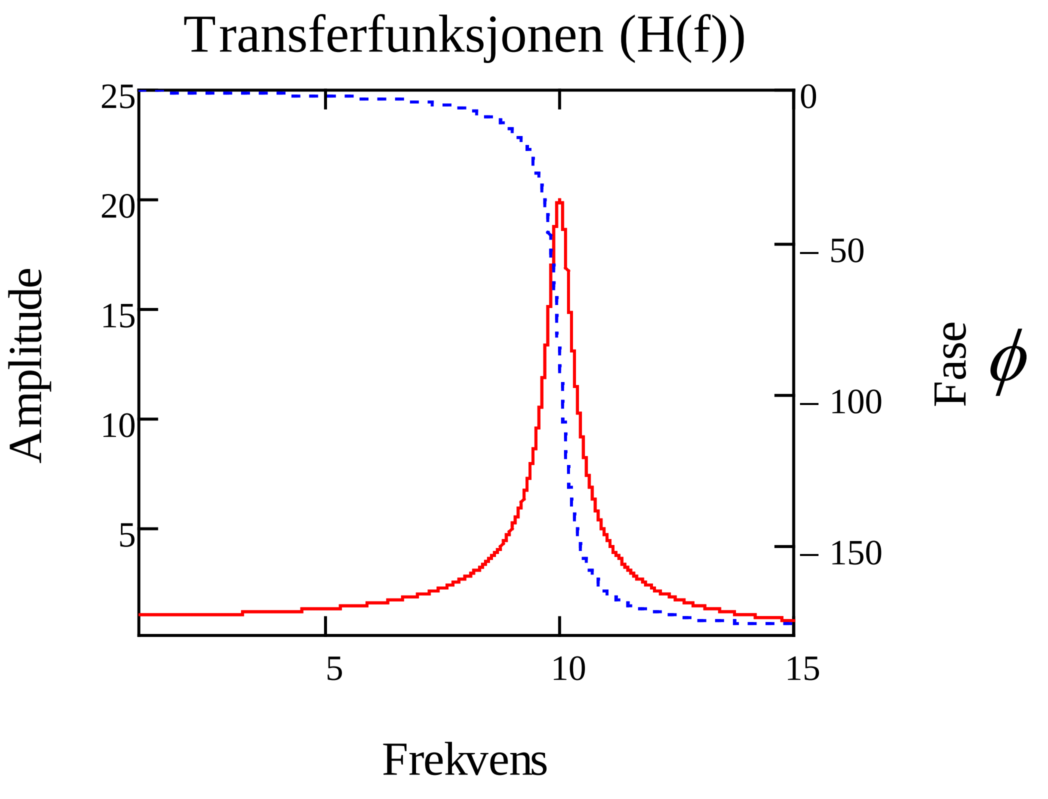
$$A_{ut} e^{j(\omega t + \phi)}$$

$$A_{ut} \cos(\omega t + \phi)$$

Da kan vi finne transferfunksjonen

$$V_{ut}(\omega) = H(\omega) V_{in}(\omega)$$

$$\frac{A_{ut}}{A_{inn}}$$



Vinkelfrekvens ω og frekvens f brukes om hverandre, ofte brukes ω i teori og f i praksis. Husk at $\omega=2\pi f$

For å beskrive et svingesystem trenger vi

- Sammenheng mellom koordinatene

$$x = \int v \, dt \qquad a = \frac{dv}{dt} \qquad q = \int i \, dt \qquad \frac{di}{dt} = \frac{di}{dt}$$

- Bevegelseslikninger

$$F = ma \qquad Q = CV \qquad V = L\frac{dI}{dt} \qquad V = RI$$

$$F = -kx \qquad F = -\gamma v$$

- Initialbetingelser og/eller pådrag

$$V(0) = 0 \qquad f = f_0 e^{j\omega t}$$

Position – velocity - acceleration

$$\omega = 2\pi f$$

Oscillatory motion

- Position $x(t) = x_0 e^{j\omega t}, \quad x_0 \cos(\omega t)$
- Velocity $v(t) = \frac{d}{dt} x_0 e^{j\omega t} = j\omega x_0 e^{j\omega t} \quad , \quad -x_0 \omega \sin(\omega t)$
- Acceleration $a(t) = \frac{d^2}{dt^2} x_0 e^{j\omega t} = -\omega^2 x_0 e^{j\omega t} \quad , -\omega^2 x_0 \cos(\omega t)$

Consequence 1:

low frequency -> measure position or velocity

high frequency -> measure acceleration

Consequence 2

- Plotting position amplitude emphasizes low frequencies
- Plotting acceleration emphasizes high frequencies

Velocity

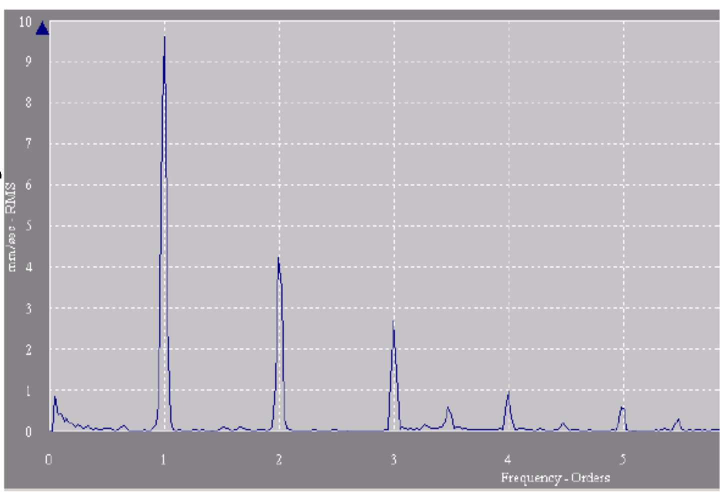


Fig 2: Misalignment Frequency

Acceleration

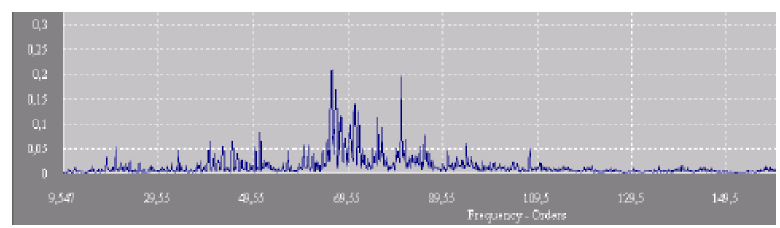
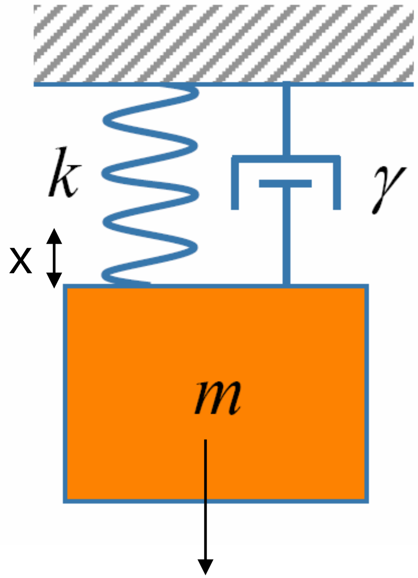


Fig 3: Bearing damage Frequency

Velocity is the usual compromise

Mekanisk svingesystem



$$f = f_0 e^{j\omega t}$$

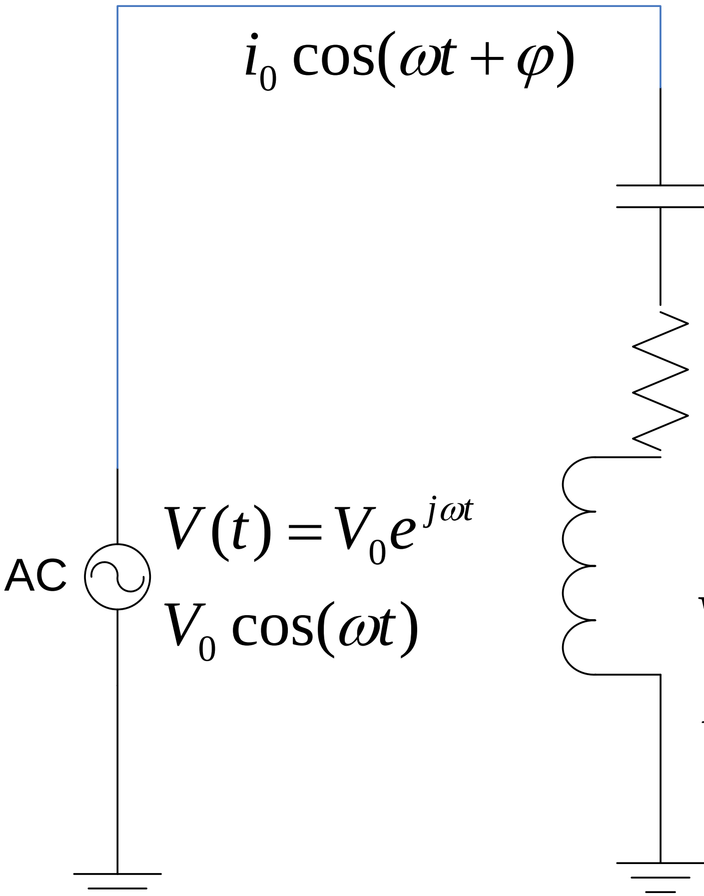
$$f = ma + \gamma v + kx$$

$$f_0 e^{j\omega t} = -m\omega^2 x_0 e^{j\omega t} + j\omega\gamma x_0 e^{j\omega t} + kx_0 e^{j\omega t}$$

$$f_0 e^{j\omega t} = \left(k + j\omega\gamma - m\omega^2\right) x_0 e^{j\omega t}$$

$$H(\omega) = \frac{x_0}{f_0} = \frac{1}{k + j\omega\gamma - m\omega^2}$$

Elektrisk svingesystem



$$i(t) = i_0 e^{j\omega t}$$

$$i_0 \cos(\omega t + \varphi)$$

$$V_C(t) = \frac{q(t)}{C} = \frac{1}{C}\int i(t)dt = \frac{1}{j\omega C} i_0 e^{j\omega t}$$

$$\frac{1}{\omega C} i_0 \sin(\omega t + \varphi)$$

$$V_R(t) = i(t)R = Ri_0 e^{j\omega t}$$

$$Ri_0 \cos(\omega t + \varphi)$$

$$V(t) = V_0 e^{j\omega t}$$

$$V_0 \cos(\omega t)$$

$$V_L(t) = L\tfrac{di}{dt} = Lj\omega i_0 e^{j\omega t}$$

$$Li_0\omega \sin(\omega t + \varphi)$$

Bevegelseslikningen

$$V_0 e^{j\omega t} = \frac{1}{j\omega C} i_0 e^{j\omega t} + R\, i_0 e^{j\omega t} + j\omega L i_0 e^{j\omega t}$$

$$V_0 e^{j\omega t} = \left(\frac{1}{C} + R\, j\omega - \omega^2 L \right) \frac{1}{j\omega} i_0 e^{j\omega t}$$

$$q(t) = \int i\, dt = \int i_0 e^{j\omega t}\, dt = \frac{1}{j\omega} i_0 e^{j\omega t}$$

$$V_0 e^{j\omega t} = \left(\frac{1}{C} + R\, j\omega - \omega^2 L \right) q_0 e^{j\omega t}$$

$$H(\omega) = \frac{q}{V_0} = \frac{1}{\frac{1}{C} + R\, j\omega - \omega^2 L}$$

Resonansfrekvens

$$H(\omega) = \frac{x_0}{f_0} = \frac{1}{k + j\omega\gamma - m\omega^2}$$

$$H(\omega) = \frac{q}{V_0} = \frac{1}{\frac{1}{C} + R\,j\omega - \omega^2 L}$$

Ved null dempning går transferfunksjonen mot uendelig når:

$$k = m\omega^2$$

$$\frac{1}{C} = L\omega^2$$

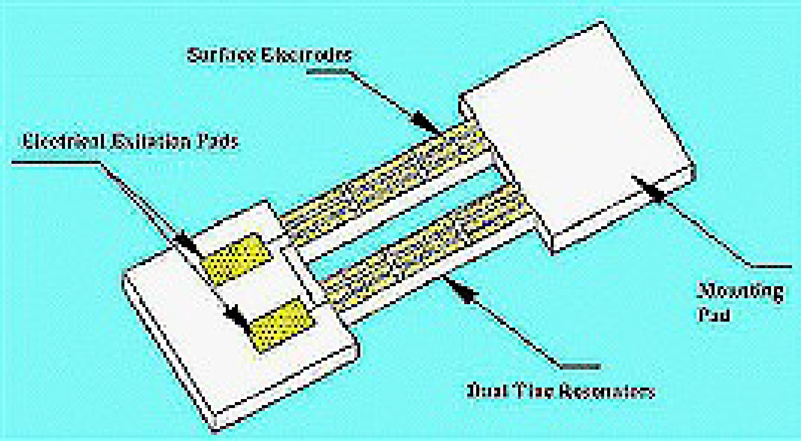
Innfører derfor udempet resonansfrekvens

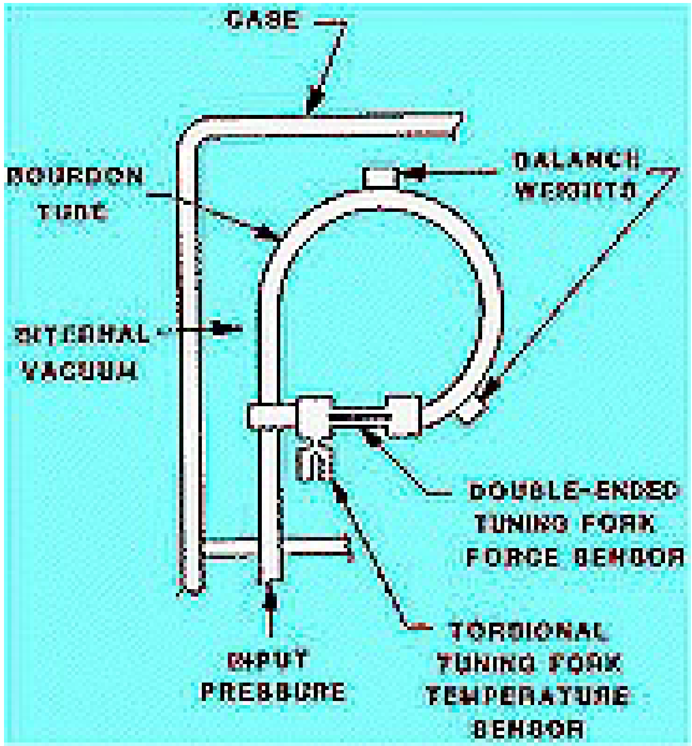
$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Quartz pressure sensor

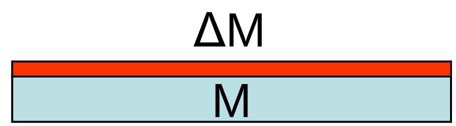
$$f = \frac{1}{2\pi}\sqrt{\frac{k + \Delta k}{M}}$$





http://www.paroscientific.com/qtechnology.htm

Quartz microbalance



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M + \Delta m}}$$

Q-faktor

$$H_a(\omega) = \frac{\frac{1}{m}}{\omega_0^{2} + \frac{j\cdot\omega_0\cdot\omega}{Q} - \omega^{2}}$$

$$H_{.}(\omega) = \frac{\frac{1}{L}}{\omega_{.0}^{2} + \frac{1j\cdot\omega_{.0}\cdot\omega}{Q} - \omega^{2}}$$

$$Q = \frac{m\cdot\omega_0}{\gamma}$$

$$Q = \frac{\omega_{.0}}{RC}$$

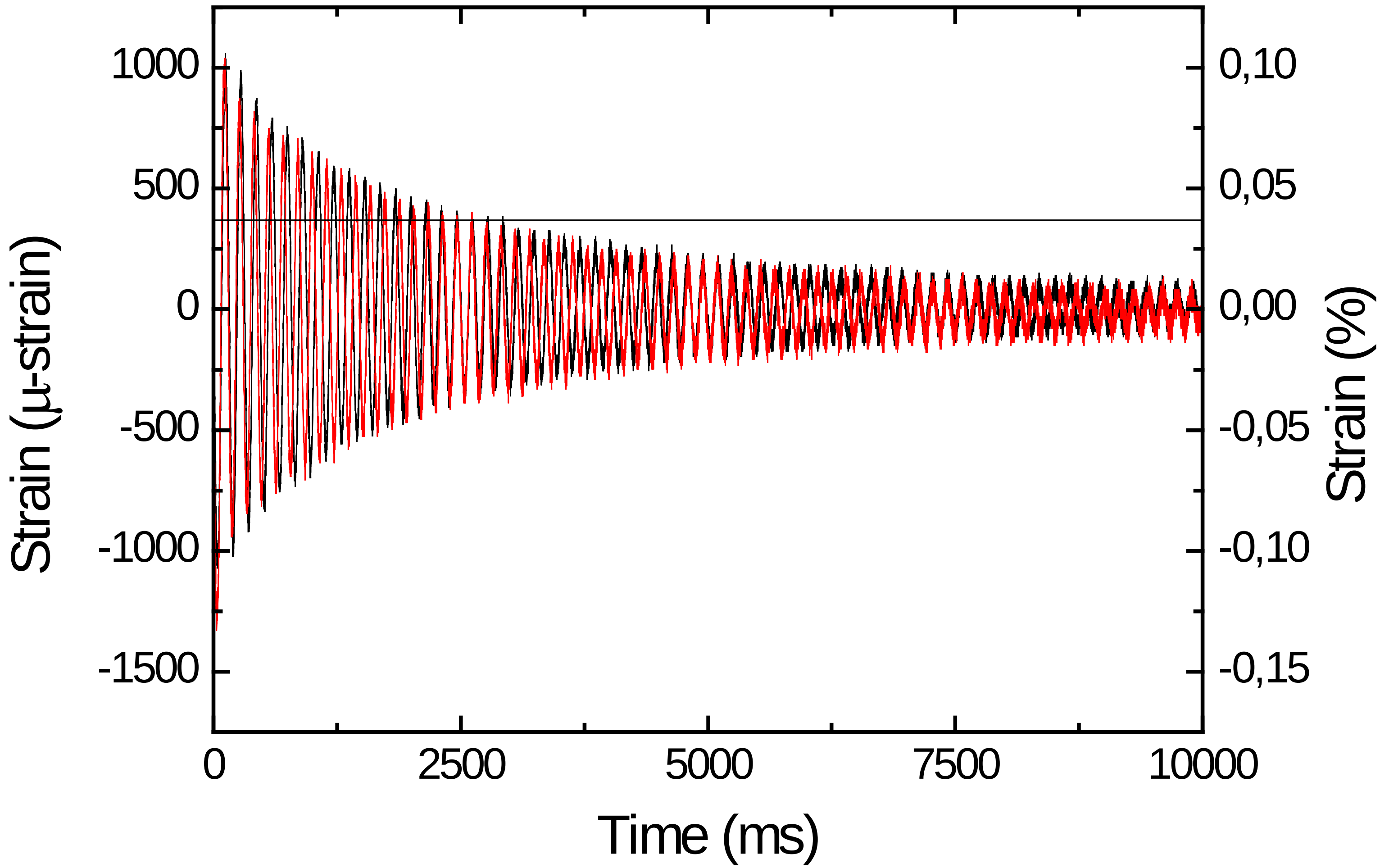
$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Q-factor appears as

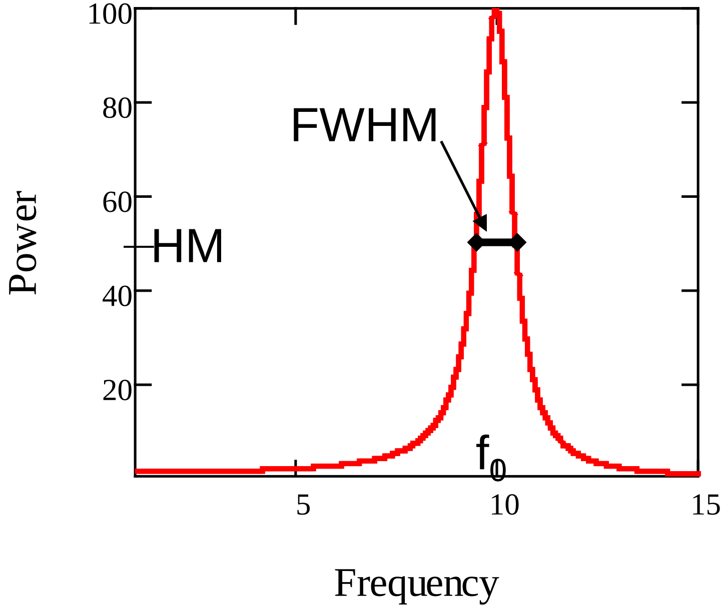
- Stored energy divided by energy dissipated during one cycle at resonance
- Number of oscillations before the amplitude is reduced by a factor 1/e
- Eigenfrequency divided by the Full Width at Half Maximum for the transfer function squared (power)

Q from decay



1000
500
0
-500
-1000
-1500
Strain (μ-strain)
0,10
0,05
0,00
-0,05
-0,10
-0,15
Strain (%)
0
2500
5000
7500
10000
Time (ms)

Q from power function

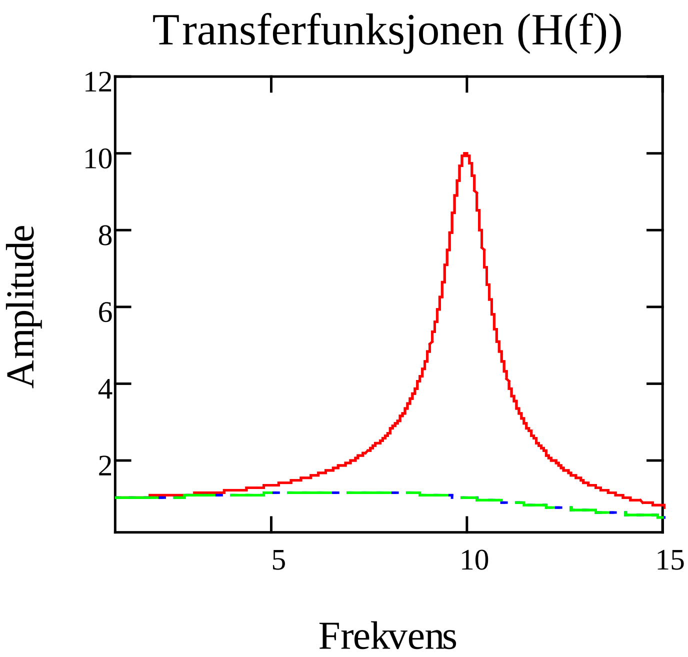


$$Q = \frac{f_0}{FWHM}$$

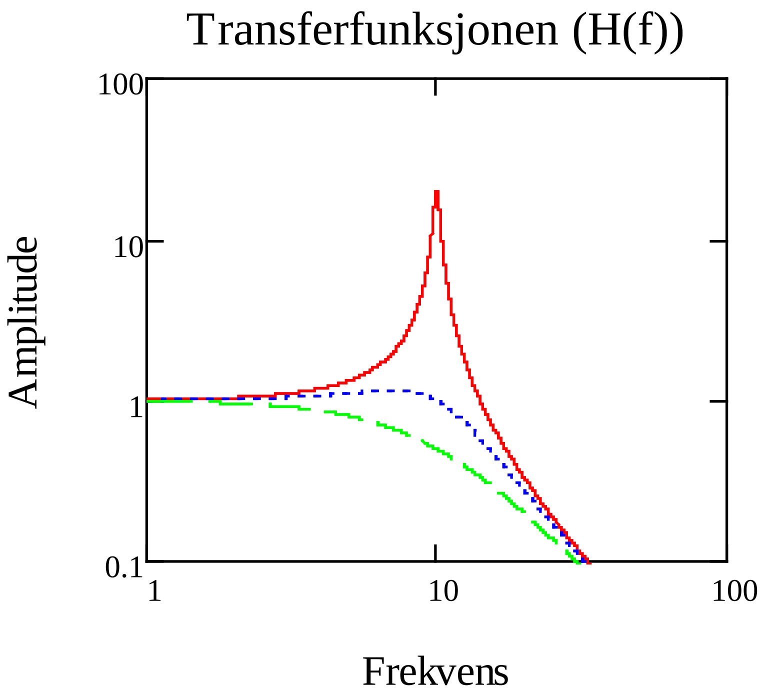
Amplitude og fase

$$|H(\omega)| = \frac{1}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \frac{\omega^2 \cdot \omega_0^2}{Q^2}}} \qquad \theta(\omega) = \operatorname{atan}\left[\frac{\omega \cdot \omega_{.0}}{Q \cdot \left(\omega^2 - \omega_{.0}^2\right)}\right]$$

Amplitude Q=1/2,1,10



Lineære akser



Logaritmiske akser

Kritisk dempning (se kompendiet): $\zeta = \frac{1}{2Q} = 1$

Resonans og fase

Arbeid utført av kraften på svingesystemet

$$W = \int F\,dx = \int F\,\frac{dx}{dt}\,dt = \int F\; j\omega\, x\, dt$$

Hvis kraft og posisjon er i fase:

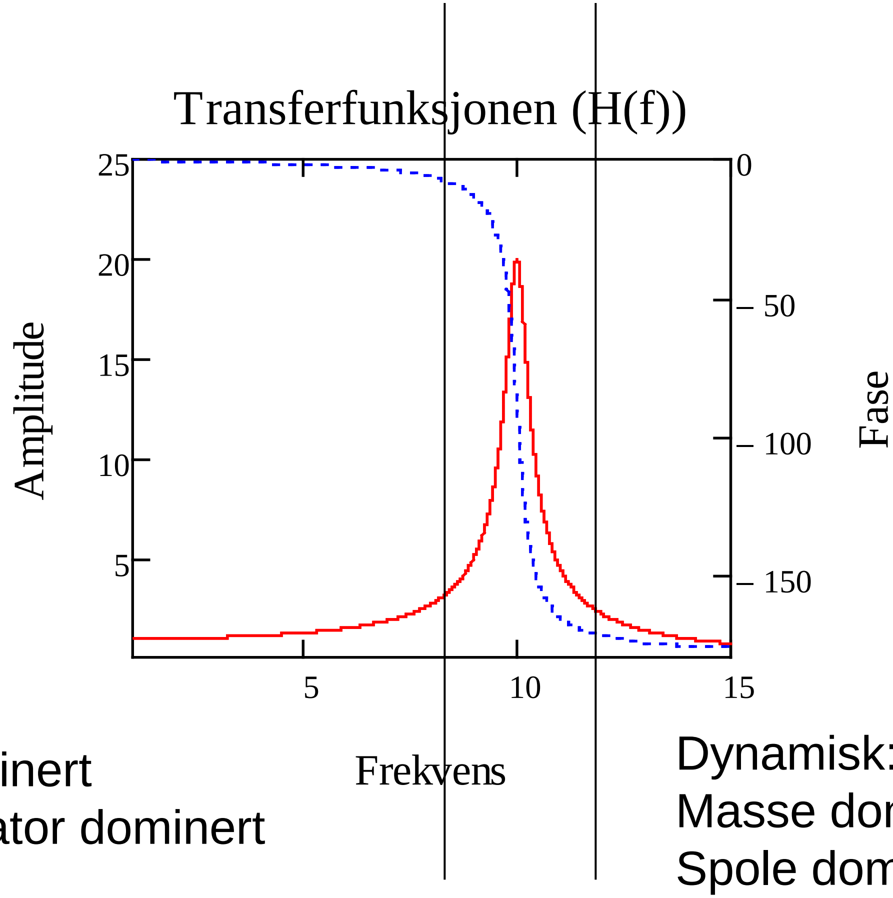
$$W = \int \mathrm{Re}(f_0 e^{j\omega t})\,\mathrm{Re}(j\omega\, x_0 e^{j\omega t})\,dt = \omega\, x_0 f_0 \int \cos(\omega t)\,\sin(\omega t)\;dt \approx 0$$

Hvis posisjonen henger 90 grader etter kraften: $x_0 = \dfrac{f_0}{j\omega_0\gamma}$

$$W = \int \mathrm{Re}(f_0 e^{j\omega t})\,\mathrm{Re}\Big(j\omega \frac{f_0}{j\omega_0\gamma} e^{j\omega t}\Big)\,dt = \frac{1}{\gamma}\, f_0^{\;2} \int \cos(\omega t)\,\cos(\omega t)\;dt \neq 0$$

Frekvens regimer

Resonans

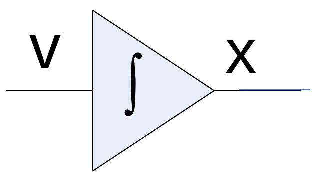


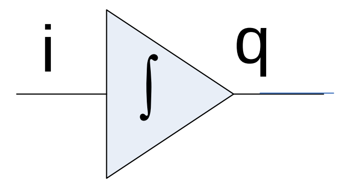
Statisk:
Fjær dominert
Kondensator dominert

Dynamisk:
Masse dominert
Spole dominert

Blokkfunksjonsbeskrivelse

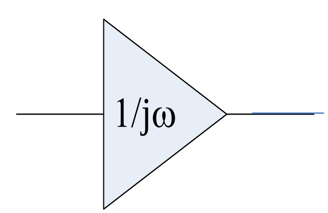
- Tegner alltid integrasjon





- Med antagelsen $f = C_1 e^{j\omega t}$
  får vi $\int f dt = \frac{1}{j\omega} C_1 e^{j\omega t} +$

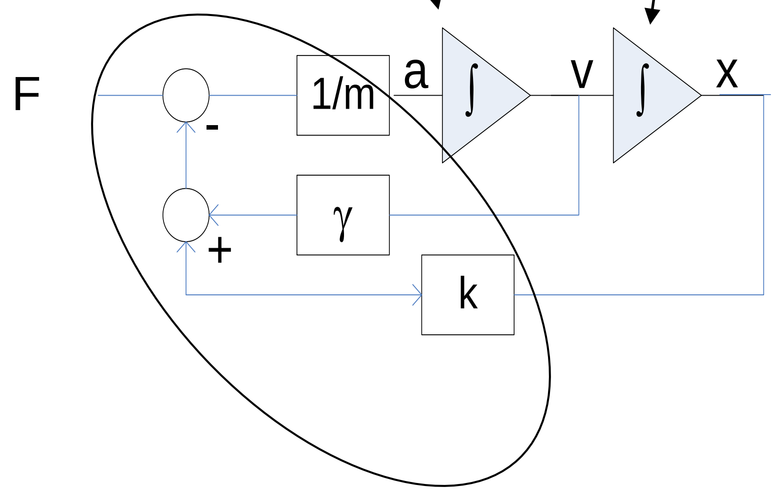
- Derfor skriver vi ofte



Blokkfunksjon for svingesystem

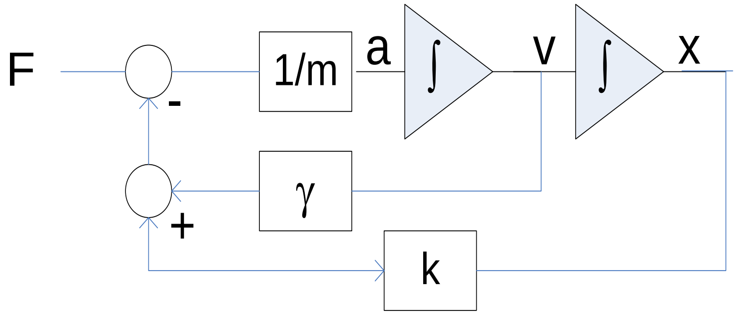
$$v = \int a \, dt$$

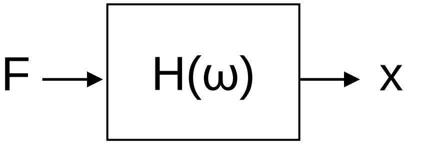
$$x = \int v \, dt$$



$$a = \frac{1}{m}\left(F - (\gamma v + kx)\right)$$

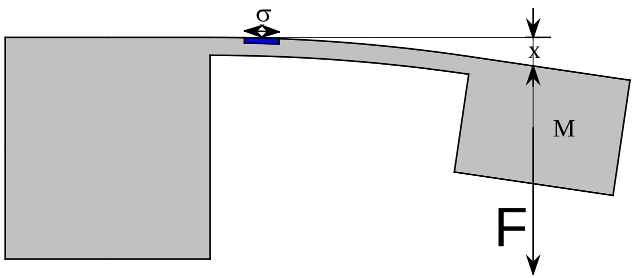
Transferfunksjonen

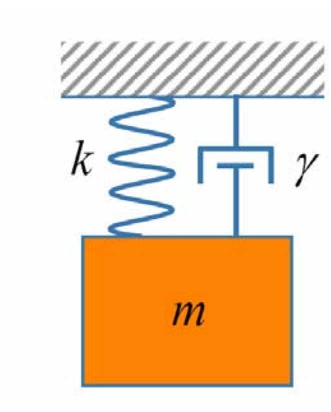


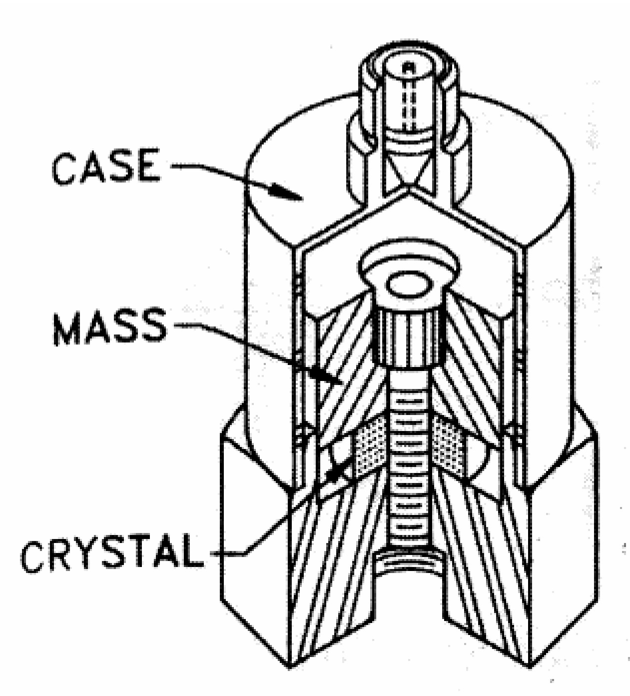


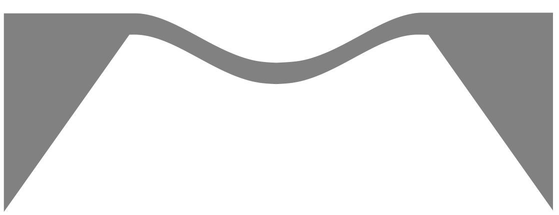
$$H(\omega) = \frac{x_0}{f_0} = \frac{1}{k + j\omega\gamma - m\omega^2}$$

Examples



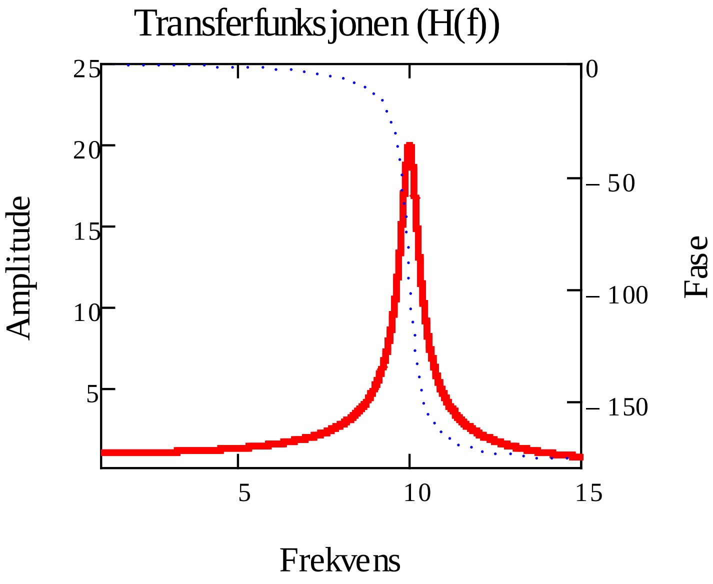






Resonerende sensorer

- Eksiterer et svingesystem
- Finner/følger resonanstoppen
  - Sveip
  - Faselåst sløyfe
  - Selvoscillerende krets
- Avlede størrelser som bestemmer resonansfrekvensen



Analogies

Table 3.4. Mechanical, Thermal, and Electrical Analogies

MECHANICAL	THERMAL	ELECTRICAL	
MASS M Kinetic energy $F=M\frac{d(v)}{dt}$	CAPACITANCE C $Q=C\frac{dT}{dt}$	INDUCTOR L Magnetic en $V=L\frac{di}{dt}$	CAPACITOR $i=C\frac{dV}{dt}$
SPRING k Potential energy $F=k\int v dt$	CAPACITANCE C $T=\frac{1}{C}\int Q dt$	CAPACITOR C Electrostatic en $V=\frac{1}{C}\int i dt$	INDUCTOR L $i=\frac{1}{L}\int V dt$
DAMPER b $F=bv$	RESISTANCE R $\Delta T=RQ$ $Q=\frac{1}{R}(T_2-T_1)$	RESISTOR R $V=Ri$	RESISTOR R $i=\frac{1}{R}V$

Force control

Voltage control

Current control