Første ordens system

- Fysikk
- Matematikk
- Blokkdiagram

Stoff fra:

- Fraden 2.16, 3.14, 14.6.4

Hva trenger vi fysisk

- Et sted å lagre energi
  - Kondensator
  - Spole
  - Masse i bevegelse (kinetisk energi)
  - Fjær
  - Potensiell energi i tyngdefeltet
  - Termisk energi i en ”klump”
- Et sted å kvitte oss med energien (gjøre den om til varme)
  - Elektrisk motstand
  - Demper
  - Varmeleder
- Kople det sammen

Hva trenger vi for beskrivelsen

- Sammenheng mellom koordinatene

$$x = \int v\, dt \qquad a = \frac{dv}{dt} \qquad q = \int i\, dt \qquad \frac{di}{dt} = \frac{di}{dt}$$

- Bevegelseslikninger

$$F = ma \qquad Q = CV \qquad V = L\frac{dI}{dt} \qquad V = RI$$

$$F = -kx \qquad F = -\gamma v$$

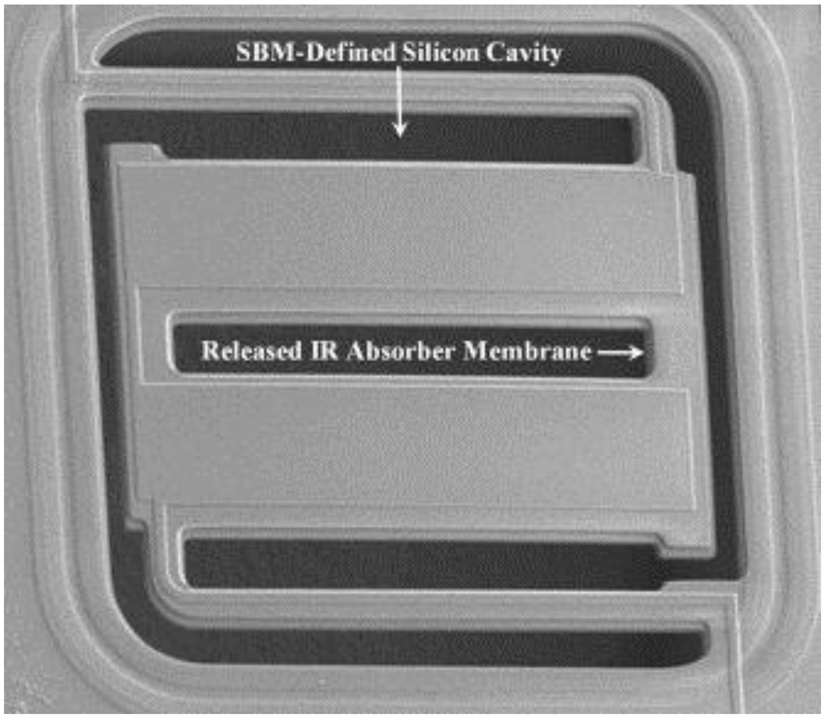
- Initialbetingelser og eventuelt pådrag

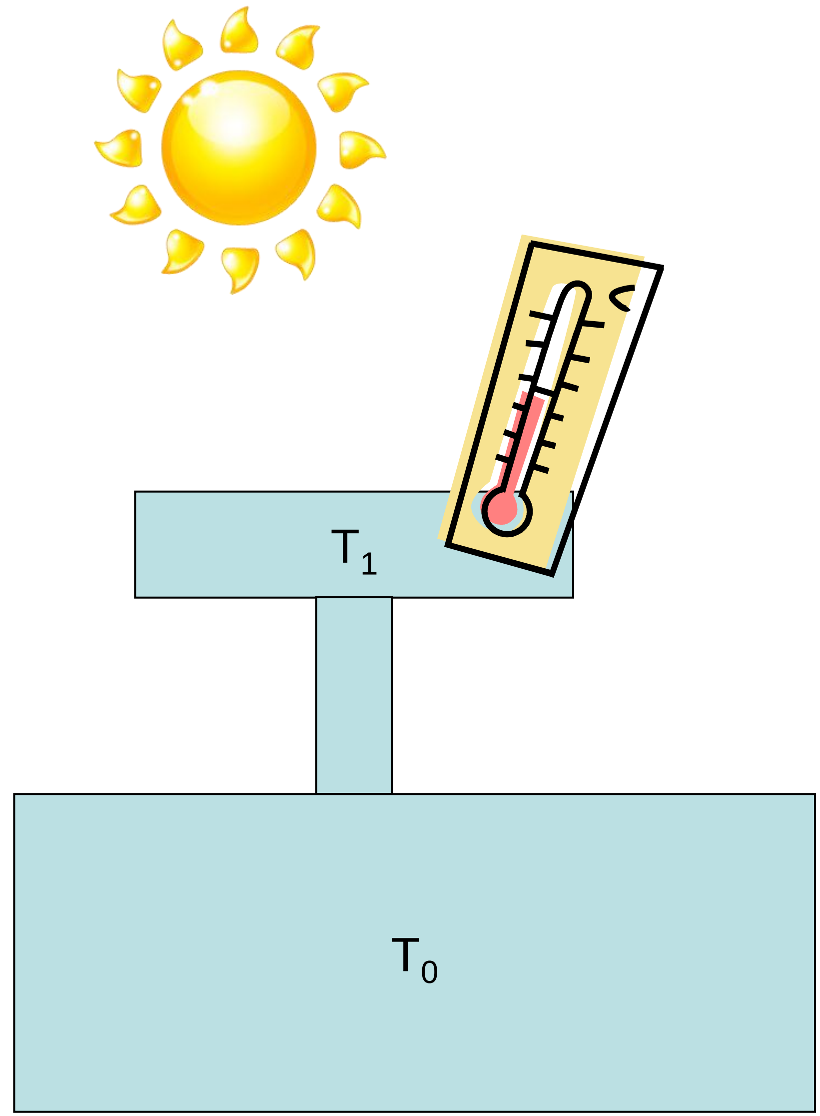
$$V(0) = 0 \qquad V(t) = 5V\sin(t)$$

Hva bruker vi for å løse likningen

- Laplace transformasjon
  - Transformere likningen og initialbetingelsene til s-planet
  - Løse likningen i s-planet
  - Transformere tilbake
- Eller:
  - Anta at løsningen er av formen: $K_0 + K_1 e^{st}$
  - Bestem koeffisientene ved direkte innsetting i bevegelseslikningen

Example: Bolometer





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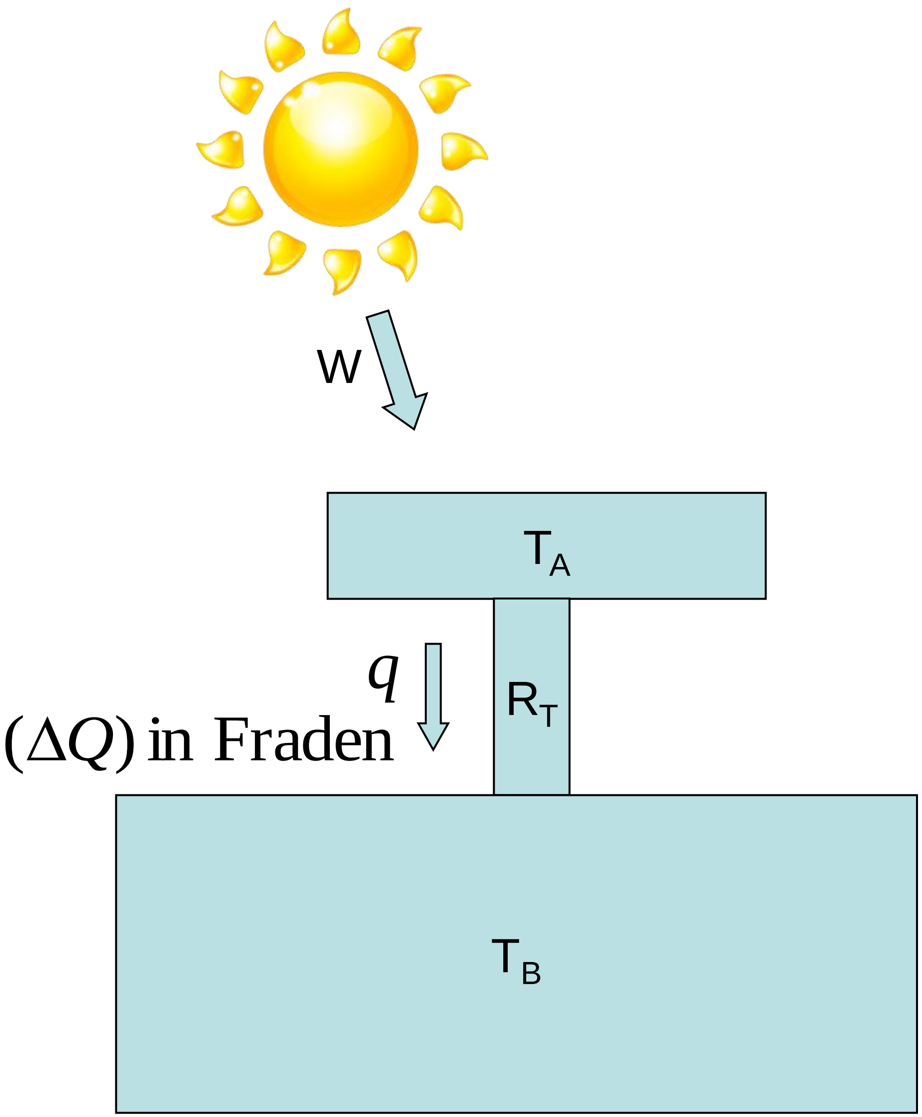
Steady state



At steady state:

$$q = W$$

Definition of thermal resistance:

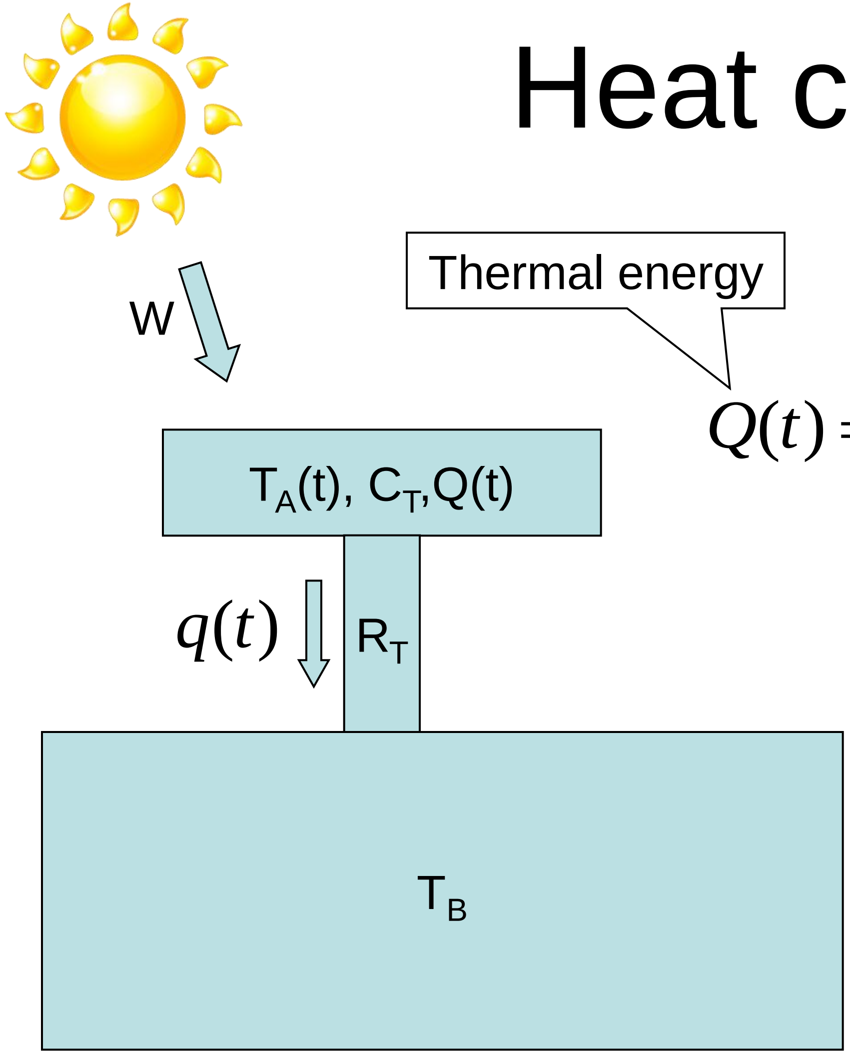
$$T_A - T_B = R_T q$$

Hence:

Maximize

$$T_A = T_B + R_T W$$

Heat capacity



Thermal energy

Heat capacity

$$Q(t) = C_T T_A(t)$$

$$\frac{\partial Q}{\partial t} = W - q$$

$$C_T \frac{\partial T_A}{\partial t} = W - \frac{1}{R}(T_A - T_B)$$

Time response

At t=0 the system has a temperature T_0, then we turn of the light (W=0)

$$\frac{\partial T_A}{\partial t} = -\frac{1}{C_T R_T}\left(T_A - T_B\right)$$

Assume: $T_A(t) = K_0 + K_1 e^{-st}$

$$-sK_1 e^{-st} = -\frac{1}{C_T R_T}\left(K_0 + K_1 e^{-st} - T_B\right)$$

$K_0 = T_B$ (from t=∞)

$$s = \frac{1}{C_T R_T}$$

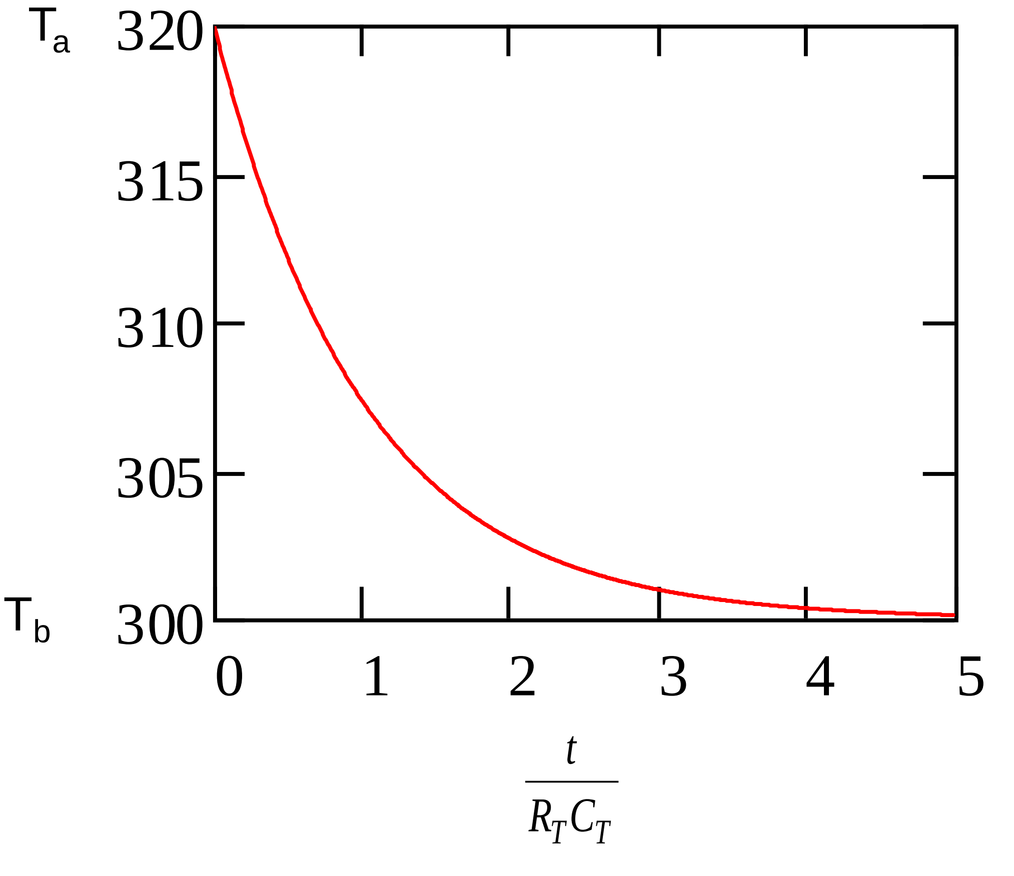
$K_1 = \left(T_A - T_B\right)$ (from t=0)

$$T_A(t) = T_B + (T_A - T_B)e^{-\frac{t}{R_T C_T}}$$

Minimize!!

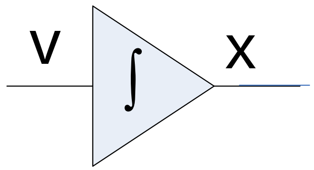
Grafisk

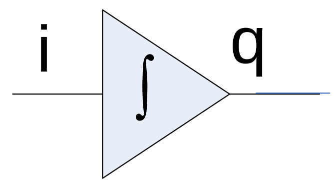
$$T_A(t) = T_B + (T_A - T_B)e^{-\frac{t}{R_T C_T}}$$



Blokkfunksjonsbeskrivelse

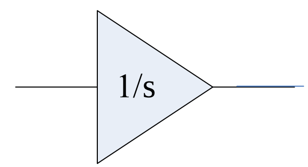
- Tegner alltid integrasjon



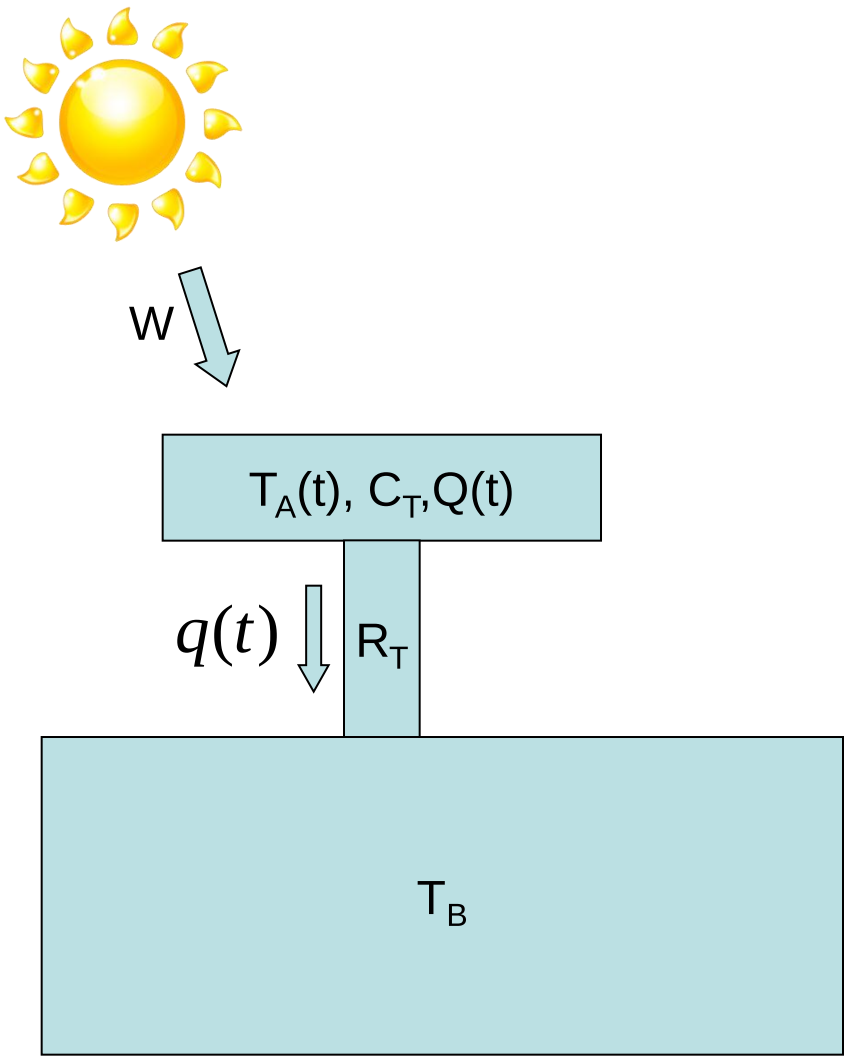


- Med antagelsen $f = C_1 e^{st}$
  får vi $\int f dt = \frac{1}{s} C_1 e^{st} +$

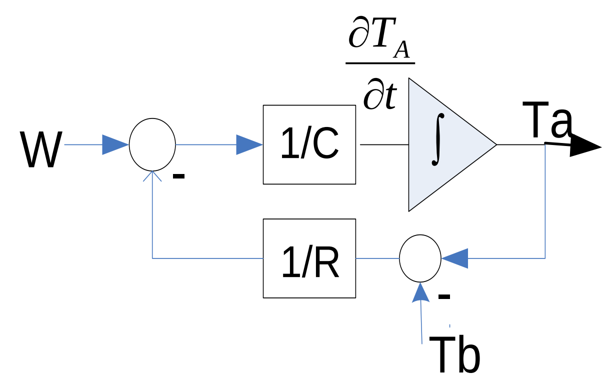
- Derfor skriver vi ofte



Forskjellige beskrivelser



$$C_T \frac{\partial T_A}{\partial t} = W - \frac{1}{R}\left(T_A - T_B\right)$$



Analogies

Table 3.4. Mechanical, Thermal, and Electrical Analogies

MECHANICAL	THERMAL	ELECTRICAL	
MASS M Kinetic energy $F=M\frac{d(v)}{dt}$	CAPACITANCE C $Q=C\frac{dT}{dt}$	INDUCTOR L Magnetic en $V=L\frac{di}{dt}$	CAPACITOR $i=C\frac{dV}{dt}$
SPRING k Potential energy $F=k\int v\,dt$	CAPACITANCE C $T=\frac{1}{C}\int Q\,dt$	CAPACITOR C Electrostatic en $V=\frac{1}{C}\int i\,dt$	INDUCTOR L $i=\frac{1}{L}\int V\,dt$
DAMPER b $F=bv$	RESISTANCE R ΔT=RQ $Q=\frac{1}{R}(T_2-T_1)$	RESISTOR R $V=Ri$	RESISTOR R $i=\frac{1}{R}V$

Force control

Voltage control

Current control

Tidskonstanter

Har sett på tidsforløp e^{st}

Det er også vanlig å skrive dette som $e^{-\frac{t}{\tau}}$

Tidskonstantene, τ ,som kan dukke opp i førsteordens system er:

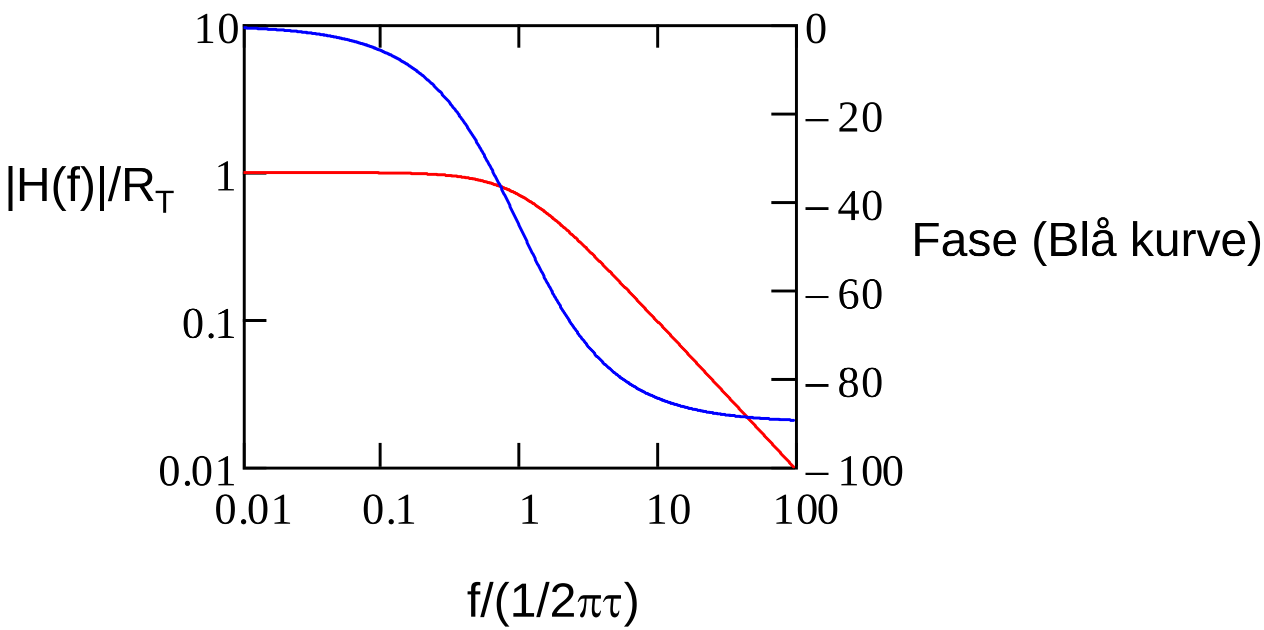
- Kondensator - motstand RC
- Spole - motstand L/R
- Masse - motstand m/γ
- Fjær – motstand γ/k
- Termisk masse – termisk motstand $R_T C_T$

Transferfunksjon

Periodisk eksitasjon $W(t) = W_{avg} + W_0 e^{j2\pi ft}$

Antar $\left(T_A - T_B\right) = \Delta T \cdot e^{j2\pi ft}$

Finner $$H\left(f\right) = \frac{\Delta T}{W_0} = \frac{R_T}{1 + j2\pi f\tau}$$



$$H\left(\frac{1}{2\pi\tau}\right) = \frac{R_T}{2}\left(1 + 1j\right)$$