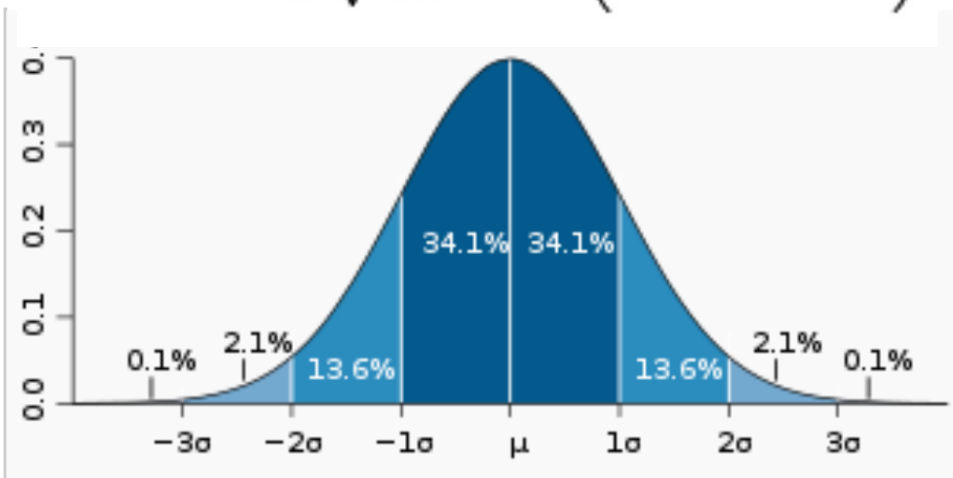
Statistiske egenskaper ved målesystemer

Stoff fra

- Kompendiet
- Fraden 2.20
- Labøvelse 1

Normalfordelingen

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



Dark blue is less than one standard deviation from the mean. For the normal distribution, this accounts for about 68% of the set (dark blue), while two standard deviations from the mean (medium and dark blue) account for about 95%, and three standard deviations (light, medium, and dark blue) account for about 99.7%.

www.wikipedia.org

Estimat av parametrene

Måler y N ganger:

Middelverdi:

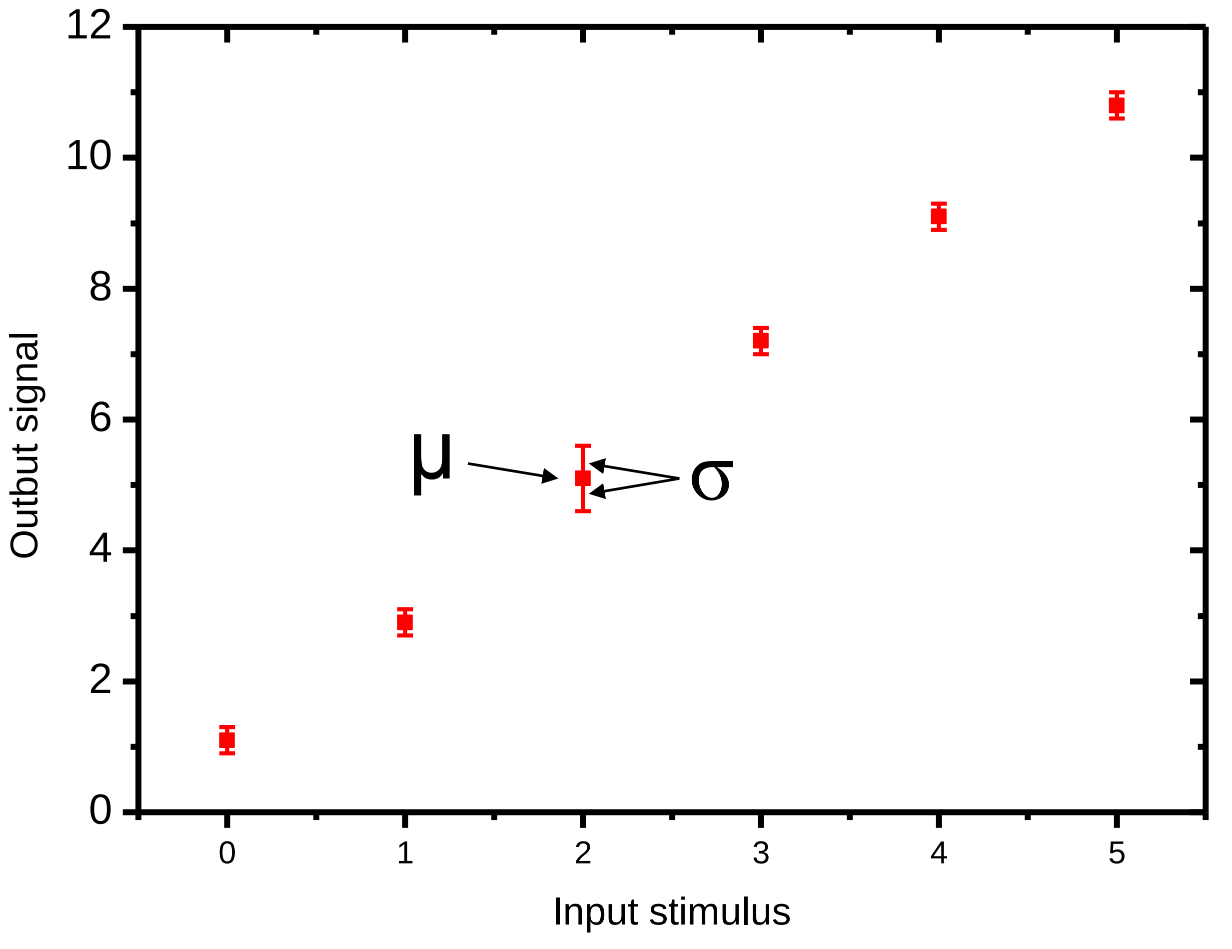
$$\mu = \frac{1}{N}\sum_{i=1}^{N} y_i = \langle y \rangle$$

Standard avvik:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \mu\right)^2}$$

$$= \sqrt{\left\langle \left(y - \langle y \rangle\right)^2 \right\rangle}$$

Fremstilling av målinger (for varierende input)



Ko-varians

Ikke korrelerte størrelser

Korrelerte størrelser

$$Cov(e_x, e_y) = \left\langle \left(e_x - \langle e_x \rangle\right)\left(e_y - \langle e_y \rangle\right) \right\rangle$$

Ukorrelerte feilkilder

Måleverdi

Feilkilde 1

$$M = S + e_x + e_y$$

Signal

Feilkilde 2

Konstant signal $\left\langle (S - \langle S \rangle)^2 \right\rangle = 0$

Tilfeldig feil $\langle e_x \rangle = \langle e_y \rangle = 0$

Ser på variansen av målesignalet

$$\left\langle (M - \langle M \rangle)^2 \right\rangle = \left\langle \left((S + e_x + e_y) - \langle (S + e_x + e_y) \rangle \right)^2 \right\rangle =$$

$$\left\langle \left((\langle S \rangle + e_x + e_y) - (\langle S \rangle + \langle e_x \rangle + \langle e_y \rangle) \right)^2 \right\rangle = \left\langle (e_x + e_y)^2 \right\rangle =$$

$$\langle e_x{}^2 \rangle \underbrace{+ 2\langle e_x e_y \rangle}_{=\text{cov}=0} + \langle e_y{}^2 \rangle$$

Usikkerhets budsjett

Table 2.2. Uncertainty Budget for Thermistor Thermometer

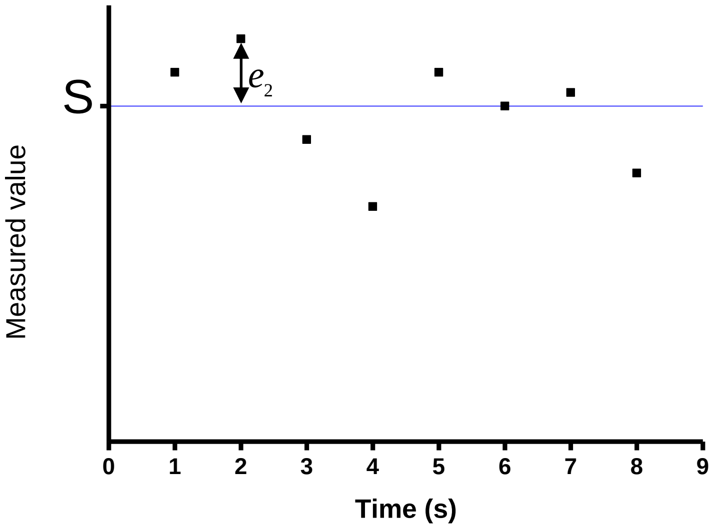
Source of Uncertainty	Standard uncertainty (°C)	Type
Calibration of sensor	0.03	B
Measured errors		
Repeated observations	0.02	A
Sensor noise	0.01	A
Amplifier noise	0.005	A
Sensor aging	0.025	B
Thermal loss through connecting wires	0.015	A
Dynamic error due to sensor's inertia	0.005	B
Temperature instability of object of measurement	0.04	A
Transmitted noise	0.01	A
Misfit of transfer function	0.02	B
Ambient drifts		
Voltage reference	0.01	A
Bridge resistors	0.01	A
Dielectric absorption in A/D capacitor	0.005	B
Digital resolution	0.01	A
Combined standard uncertainty	**0.068**	

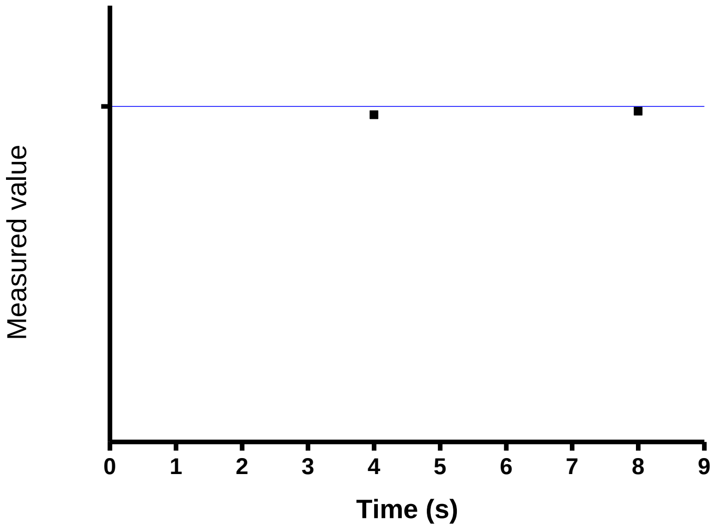
$$u_c = \sqrt{u_1^2 + u_2^2 + \cdots + u_i^2 + \cdots + u_n^2}, \qquad (2.28)$$

A: Those evaluated by statistical methods

B: Those evaluated by other means.

Midling





$$M_i = S + e_i$$

$$\left\langle \left(M - \langle M \rangle\right)^2 \right\rangle = \langle e^2 \rangle$$

$$M = \frac{1}{4}\sum_{i=1}^{4} M_i = \frac{1}{4}\sum_{i=1}^{4} S + e_i$$

$$\left\langle \left(M - \langle M \rangle\right)^2 \right\rangle = ?$$

Regner ut

$$\left\langle \left(M - \langle M \rangle\right)^2 \right\rangle = \left\langle \left(\left(\frac{1}{4}\sum_{i=1}^{4} S + e_i \right) - \left\langle \left(\frac{1}{4}\sum_{i=1}^{4} S + e_i \right) \right\rangle \right)^2 \right\rangle =$$

$$\frac{1}{16}\left\langle \left(\left(4\langle S \rangle + e_1 + e_2 + e_3 + e_4\right) - \left(4\langle S \rangle + \langle e_1 \rangle + \langle e_2 \rangle + \langle e_3 \rangle + \langle e_4 \rangle\right) \right)^2 \right\rangle = \frac{1}{16}\left\langle \left(e_1 + e_2 + e_3 + e_4\right)^2 \right\rangle$$

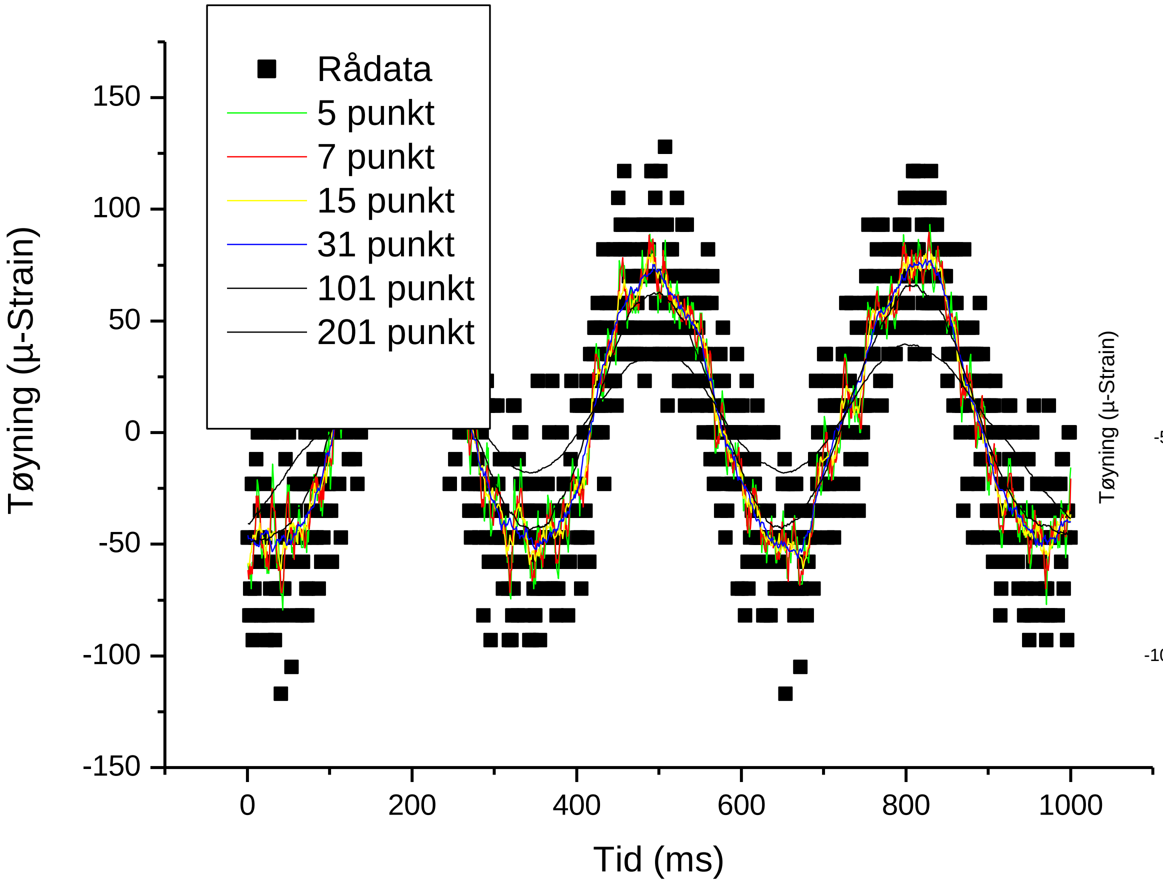
$$= \frac{1}{16}\Big(\underbrace{\langle e_1^2 \rangle + \langle e_2^2 \rangle + \langle e_3^2 \rangle + \langle e_4^2 \rangle}_{=\langle e^2 \rangle} + \underbrace{2\langle e_1 e_2 \rangle + \cdots + 2\langle e_3 e_4 \rangle}_{=\text{cov}=0} \Big) = \underline{\underline{\frac{1}{4}\langle e^2 \rangle}}$$

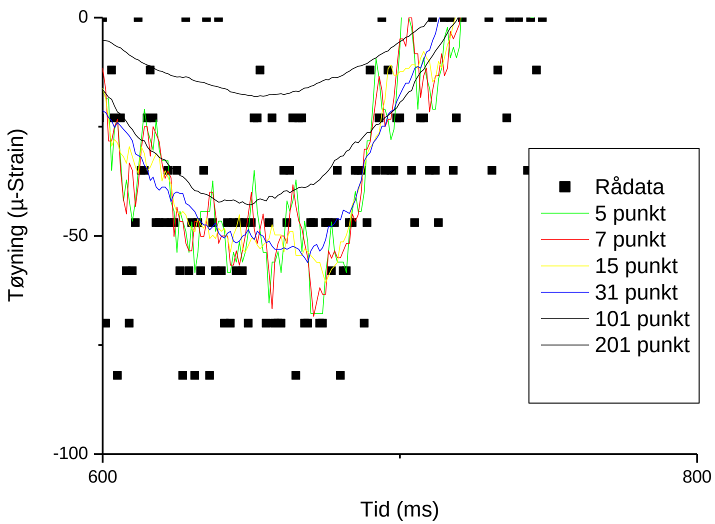
Midling over 4 punkter reduserer **standard avviket** med en faktor 2

Lett å se at midling over n punkter reduserer **standard avviket** med en faktor √n

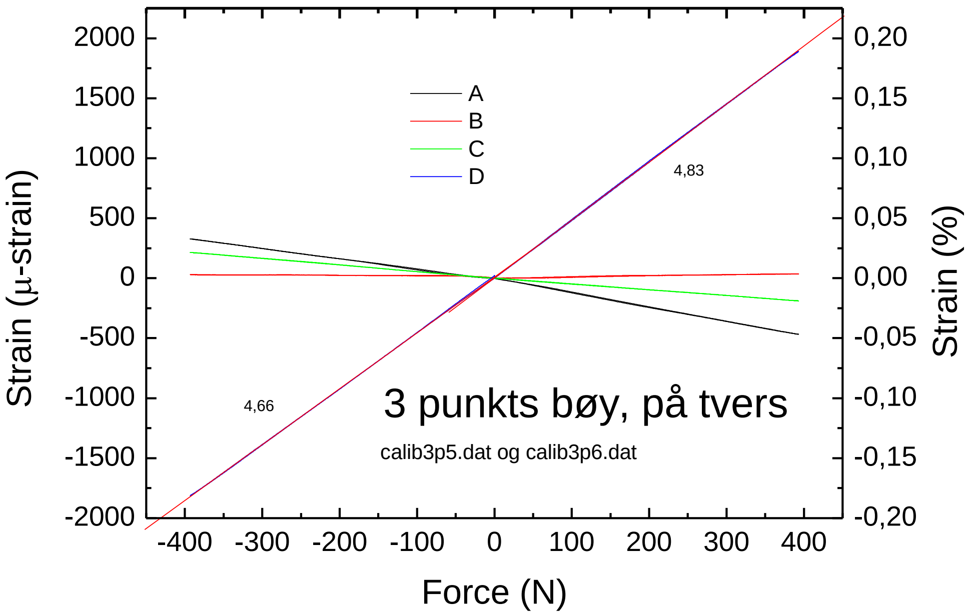
Glatting - adjacent average

$$y(i) = \frac{1}{2n+1}\sum_{n'=-n}^{n} y(i+n')$$





Kalibrering av ishockeykølle



Minste kvadraters metode

- Definerer avvik mellom modell og måling som

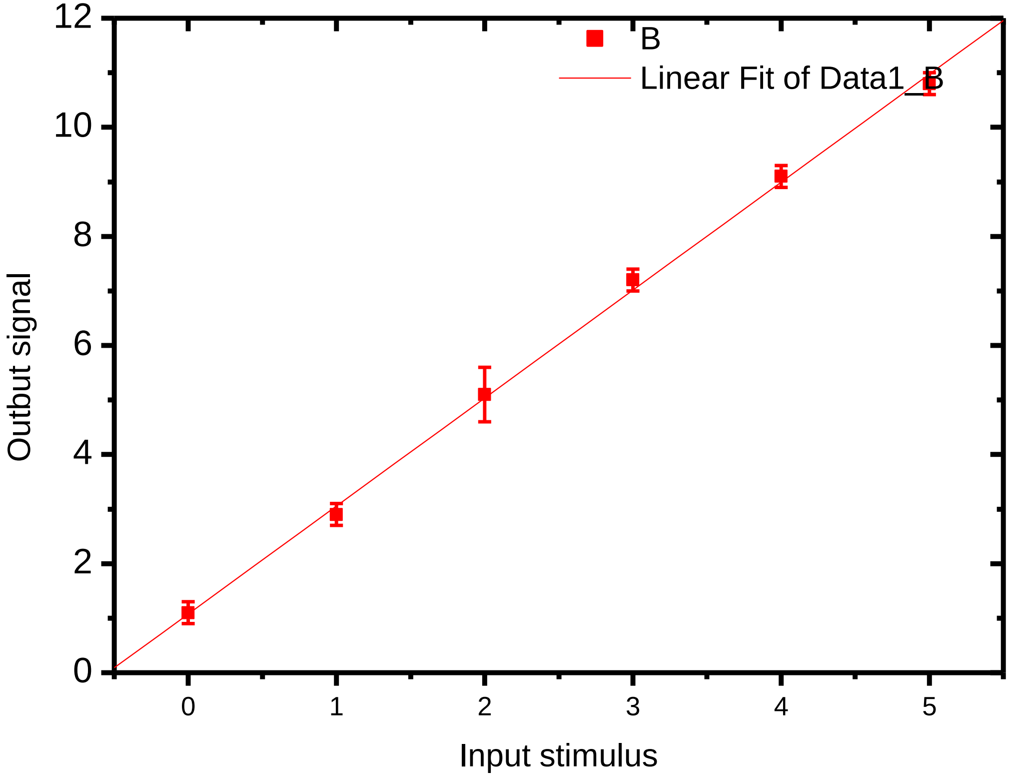
$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - y(x_i, a_1, \ldots a_M)}{\sigma_i} \right)^2$$

- Minimerer avviket med hensyn på parametrene i modellen ($a_1 \ldots\ldots a_n$)
- Lineær tilpasning hvis y er lineært avhengig av a

$$y(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_M x^{M-1}$$

=> Kan utlede formler for parametrene

Lineær regresjon med rett linje

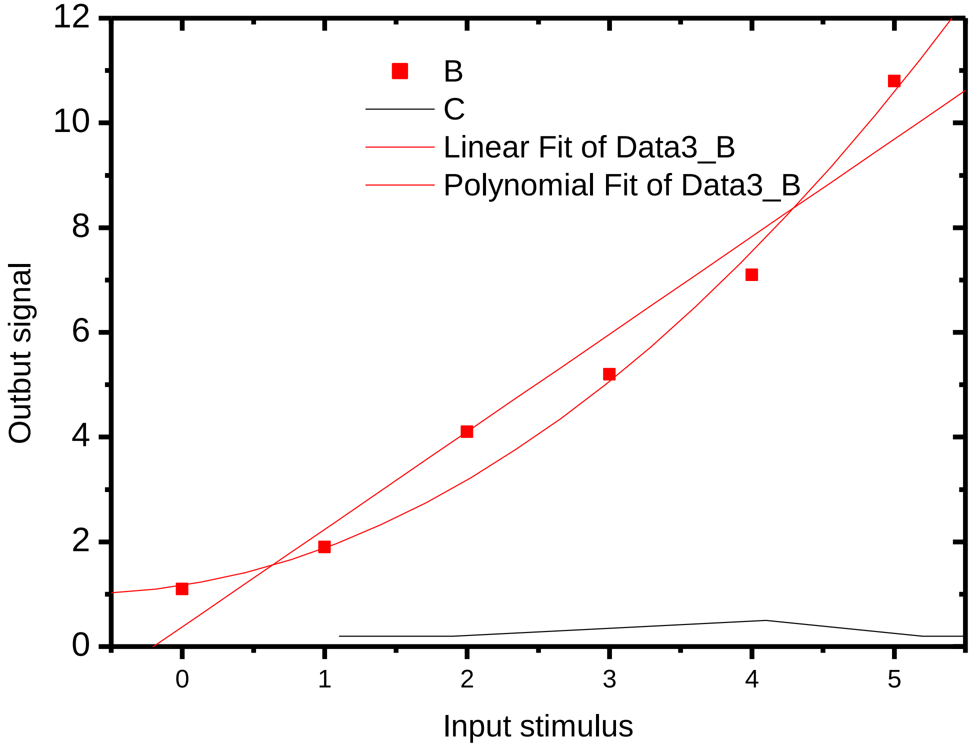


Y = A + B * X
Weight given by Data1_C error bars.

Parameter	Value	Error
A	1,07745	0,15231
B	1,97871	0,04815

R	SD	N
0,99925	0,79517	6

Lineær regresjon med første og andre orden polynom



Forventet chi-kvadrat: Antall målepunkter minus antall frihetsgrader

Savitzky-Golay

{-3, 12, 17, 12, -3} / 35

{1, 1, 1, 1, 1} / 5

- Beste tilpasning til et polynom rundt punktet man ser på

Ulineær kurvetilpasning

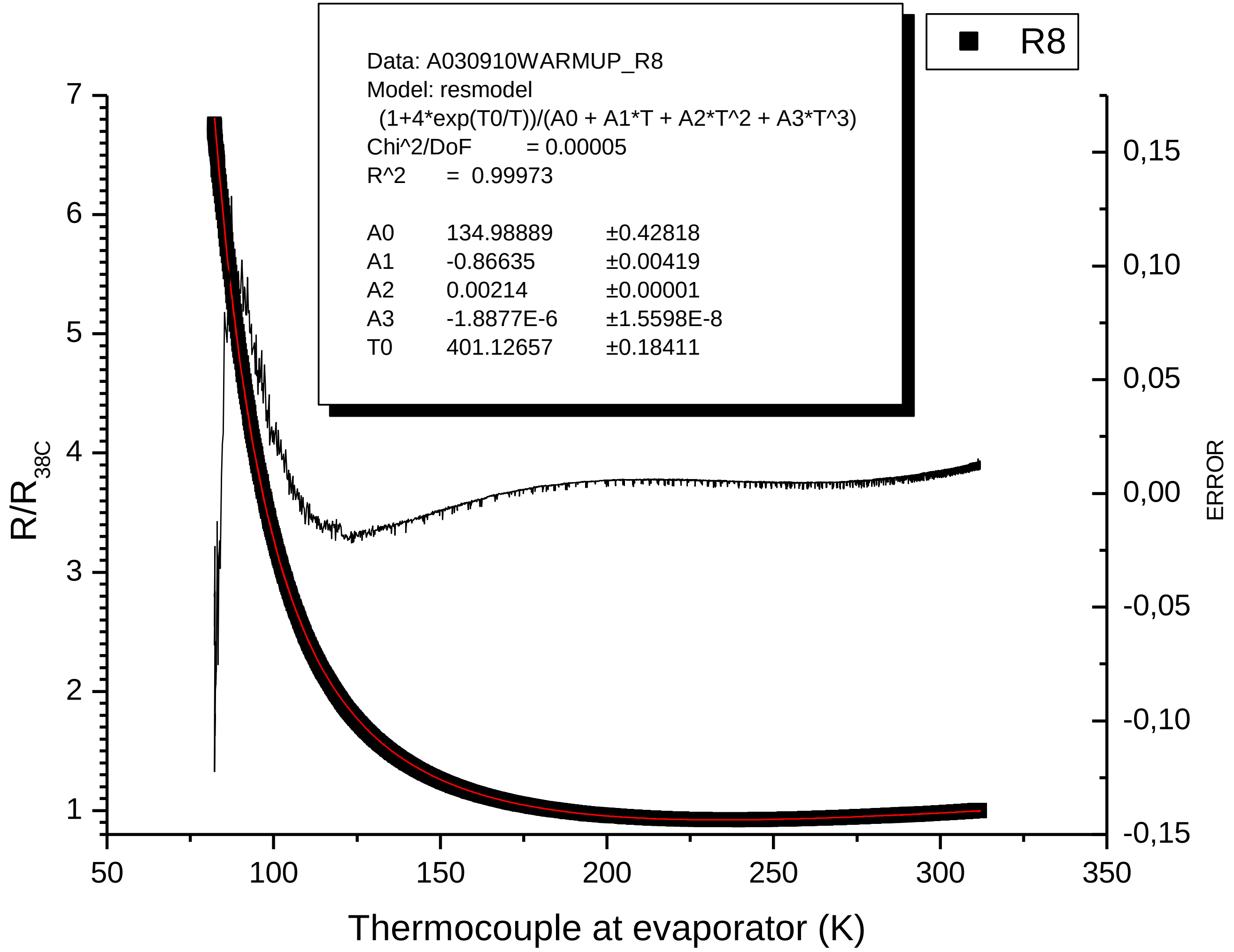
- Ulineære funksjoner $y = ae^{-\left(\frac{x-b}{c}\right)^2}$

=>ingen formler for parametrene.

- Chi-kvadrat regnes ut numerisk og minimeres med hensyn på parametrene

- Ofte en iterativ prosess som krever gjetning og overvåking av feil

Ulineær tilpasning og feil



Ikke normalfordelte avvik