

Andre ordens system og vibrasjoner

- Hvordan måle
- Hvordan sette opp en modell
 - Sidespor – vibrasjoner
- Transferfunksjon
 - Elektrisk
 - Mekanisk
- Resonerende sensorer
- Section 3.14: Dynamic Models (Fraden)
- Section 8: Velocity and acceleration (Fraden)
- Section 9.3: Piezoelectric force sensors
- Section 17.4.4: Chemical sensors

Transferfunksjon og impedans

I mekanikk er

- Transferfunksjonen forholdet mellom posisjon og kraft
- Impedans forholdet mellom hastighet og kraft

I elektronikk er (ofte)

- Transferfunksjonen forholdet mellom inngang og utgang
- Impedans forholdet mellom spenningen over kretsen og strømmen gjennom den

Fraden bruker transferfunksjon om den statiske karakteristikken til en sensor

For et svingesystem (andre ordens system) trenger vi

- To steder å lagre energi
 - Kondensator (elektrostatisk energi) og spole (magnetisk energi)
 - Masse i bevegelse (kinetisk energi) og fjær (potensiell energi)
- I tillegg har vi vanligvis et sted der energien blir gjort om til varme
 - Motstand
 - Demper

For å analysere et (svinge) system kan vi bruke

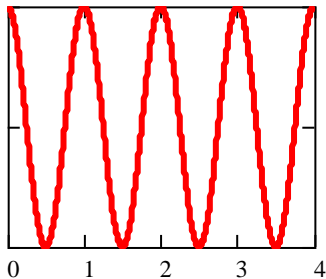
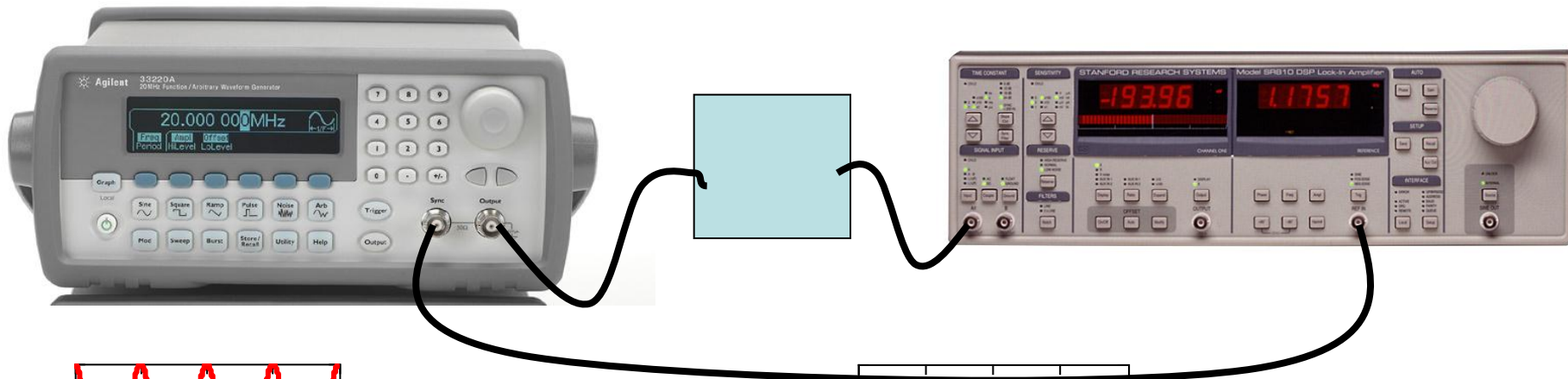
Signalgiver som gir et pådrag med bestemt

- (fase)
- frekvens
- amplitude

Analysator som kan finne

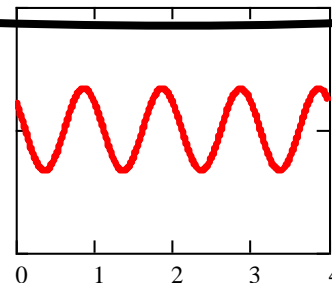
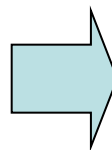
- fase
- amplitude
- (frekvens)

Av utgangssignalet



$$A_{inn} e^{j\omega t}$$

$$A_{inn} \cos(\omega t)$$



$$A_{ut} e^{j(\omega t + \phi)}$$

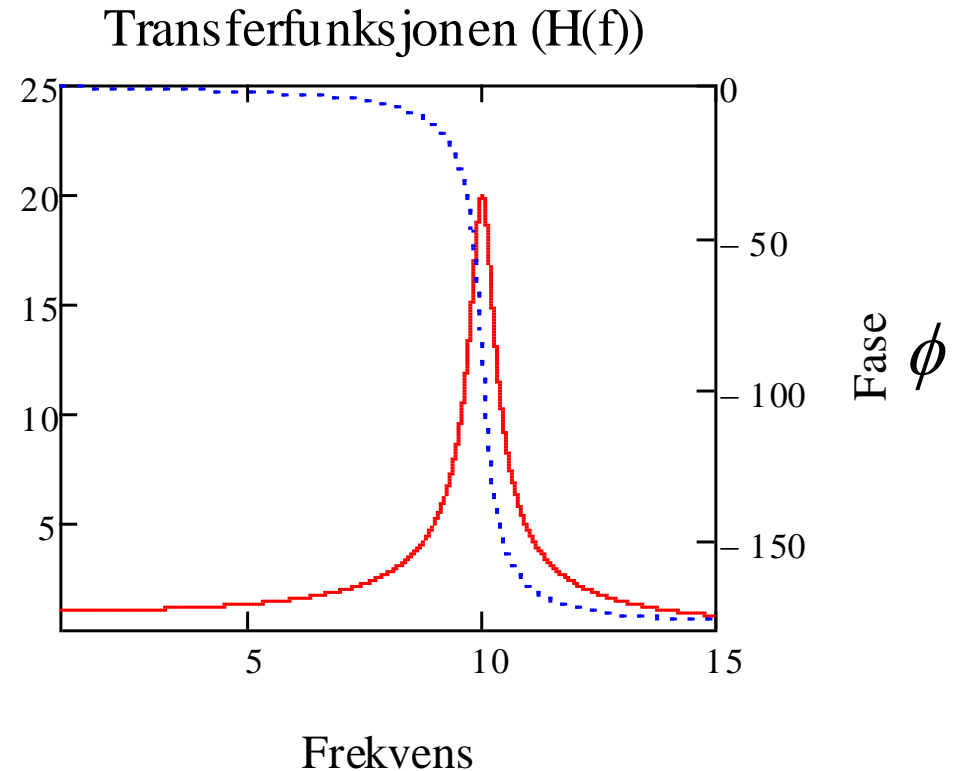
$$A_{ut} \cos(\omega t + \phi)$$

Da kan vi finne transferfunksjonen

$$V_{ut}(\omega) = H(\omega)V_{in}(\omega)$$

$$\frac{A_{ut}}{A_{inn}}$$

Amplitude



Vinkelfrekvens ω og frekvens f brukes om hverandre, ofte brukes ω i teori og f i praksis. Husk at $\omega = 2\pi f$

For å beskrive et svingesystem trenger vi

- Sammenheng mellom koordinatene

$$x = \int v \, dt \quad a = \frac{dv}{dt} \quad q = \int i \, dt \quad \frac{di}{dt} = \frac{di}{dt}$$

- Bevegelseslikninger

$$F = ma \quad Q = CV \quad V = L \frac{dI}{dt} \quad V = RI$$

$$F = -kx \quad F = -\gamma v$$

- Initialbetingelser og/eller pådrag

$$V(0) = 0 \quad f = f_0 e^{j\omega t}$$

Position – velocity - acceleration

Oscillatory motion

$$\omega = 2\pi f$$

- Position

$$x(t) = x_0 e^{j\omega t}$$

- Velocity

$$v(t) = \frac{d}{dt} x_0 e^{j\omega t} = j\omega x_0 e^{j\omega t}$$

- Acceleration

$$a(t) = \frac{d^2}{dt^2} x_0 e^{j\omega t} = -\omega^2 x_0 e^{j\omega t}$$

Consequence 1:

low frequency -> measure position or velocity

high frequency -> measure acceleration

Consequence 2

- Plotting position amplitude emphasizes low frequencies
- Plotting acceleration emphasizes high frequencies

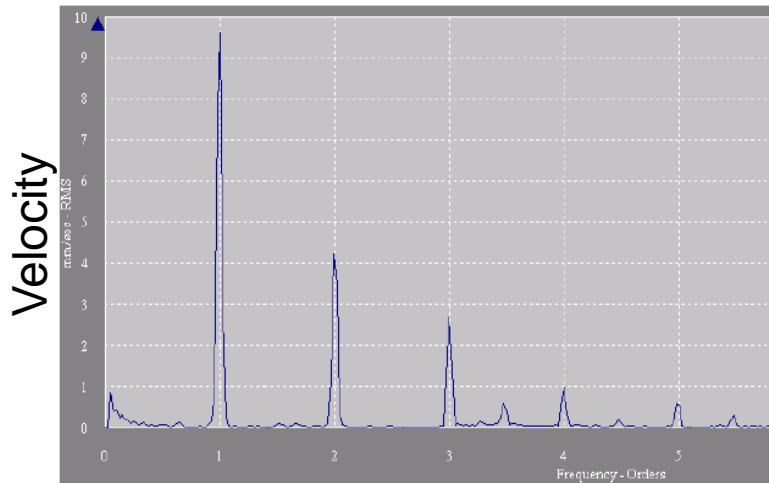


Fig 2: Misalignment Frequency

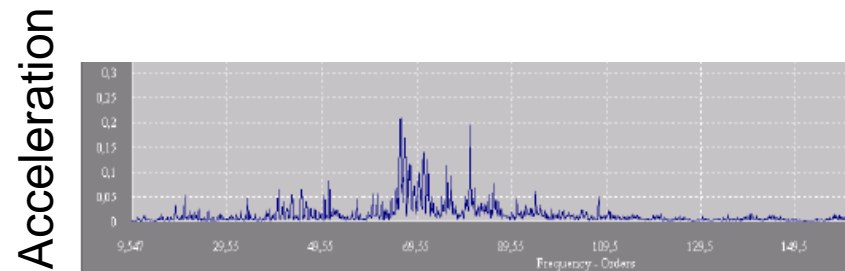
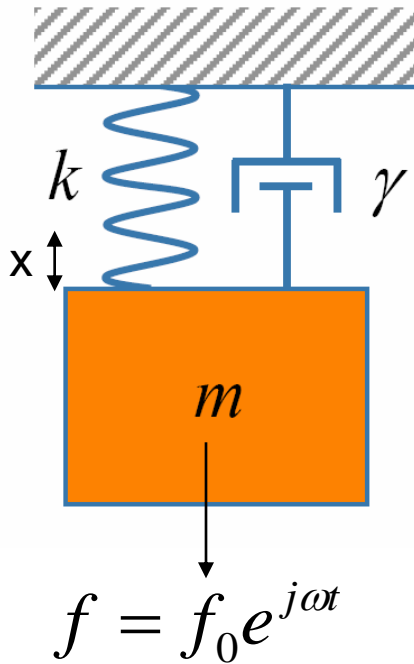


Fig 3: Bearing damage Frequency

Velocity is the usual compromise

Mekanisk svingesystem



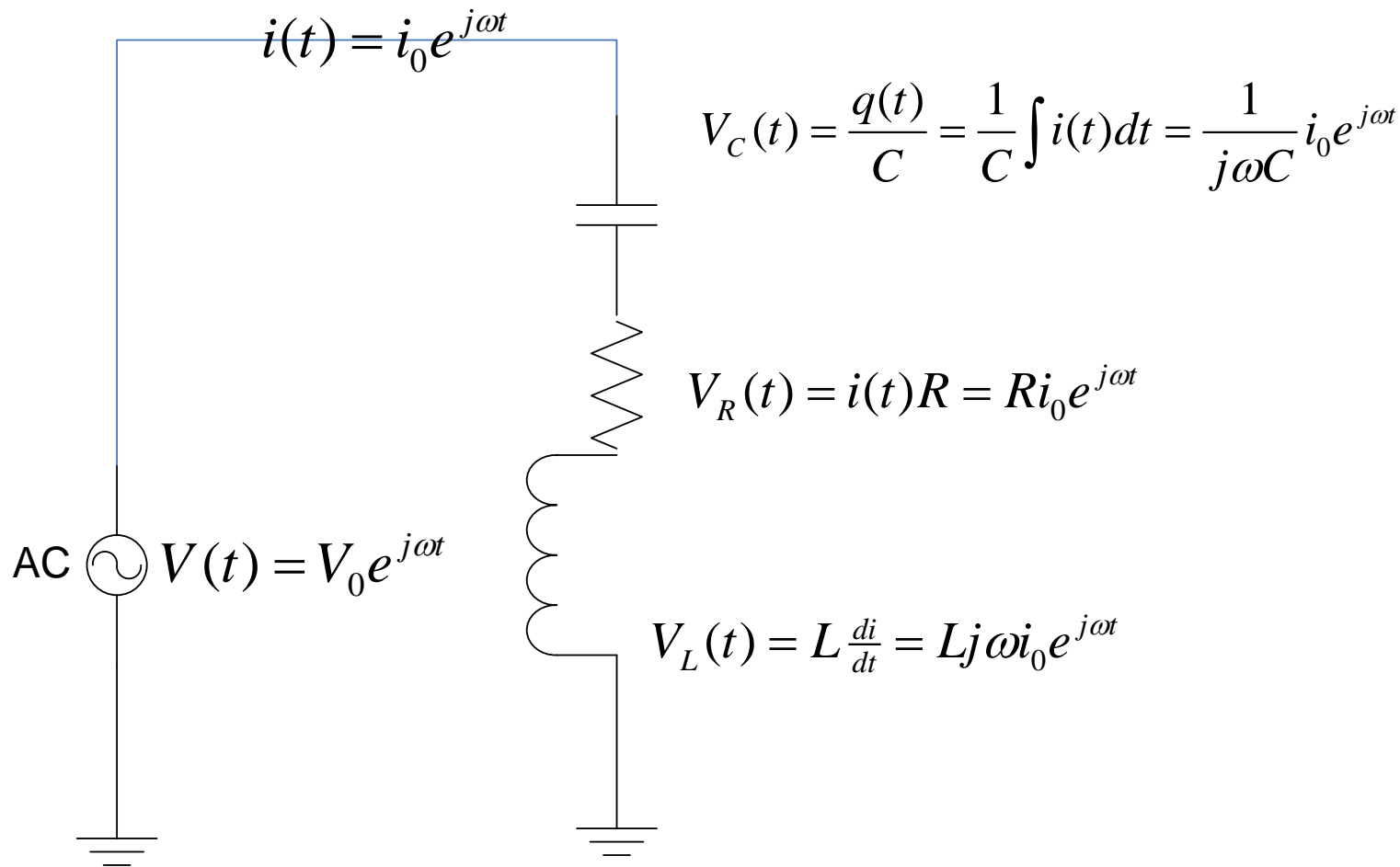
$$f = ma + \gamma v + kx$$

$$f_0 e^{j\omega t} = -m\omega^2 x_0 e^{j\omega t} + j\omega\gamma x_0 e^{j\omega t} + kx_0 e^{j\omega t}$$

$$f_0 e^{j\omega t} = (k + j\omega\gamma - m\omega^2) x_0 e^{j\omega t}$$

$$H(\omega) = \frac{x_0}{f_0} = \frac{1}{k + j\omega\gamma - m\omega^2}$$

Elektrisk svingesystem



Bevegelseslikningene

$$V_0 e^{j\omega t} = \frac{1}{j\omega C} i_0 e^{j\omega t} + R i_0 e^{j\omega t} + j\omega L i_0 e^{j\omega t}$$

$$V_0 e^{j\omega t} = \left(\frac{1}{C} + R j\omega - \omega^2 L \right) \frac{1}{j\omega} i_0 e^{j\omega t}$$

$$q(t) = \int i dt = \int i_0 e^{j\omega t} dt = \frac{1}{j\omega} i_0 e^{j\omega t}$$

$$V_0 e^{j\omega t} = \left(\frac{1}{C} + R j\omega - \omega^2 L \right) q_0 e^{j\omega t}$$

$$H(\omega) = \frac{q}{V_0} = \frac{1}{\frac{1}{C} + R j\omega - \omega^2 L}$$

Resonansfrekvens

$$H(\omega) = \frac{x_0}{f_0} = \frac{1}{k + j\omega\gamma - m\omega^2}$$

$$H(\omega) = \frac{q}{V_0} = \frac{1}{\frac{1}{C} + Rj\omega - \omega^2 L}$$

Ved null dempning går transferfunksjonen mot uendelig når:

$$k = m\omega^2$$

$$\frac{1}{C} = L\omega^2$$

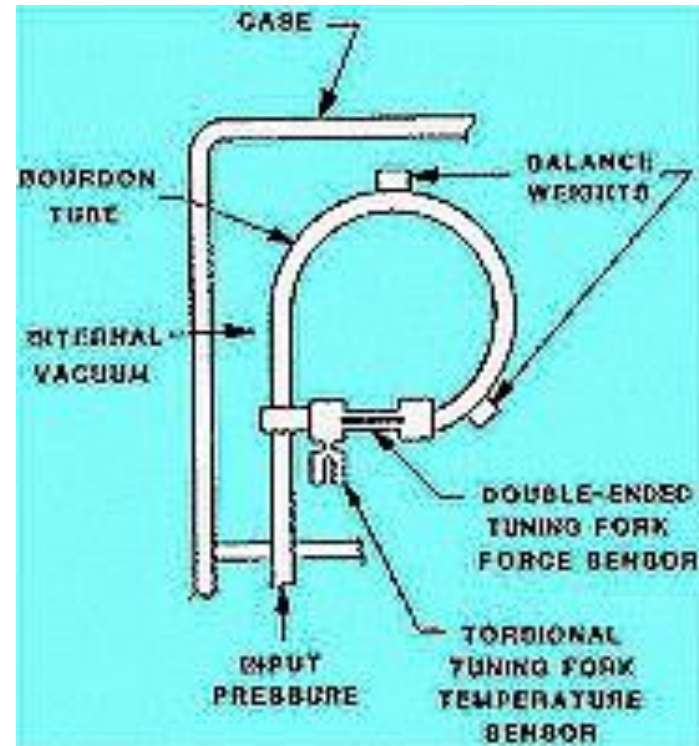
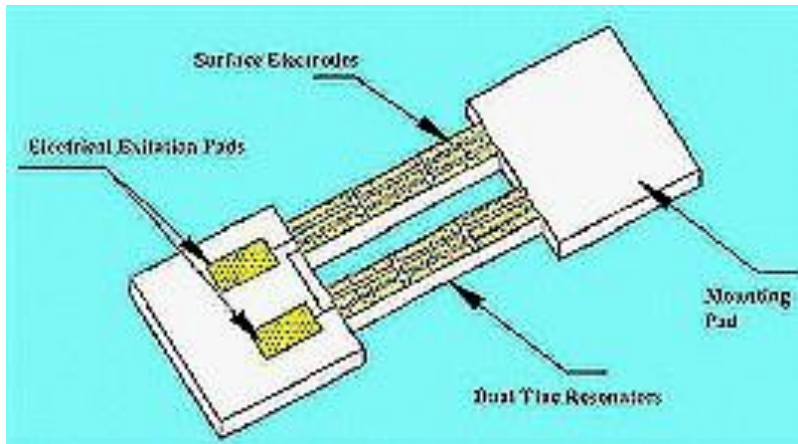
Innfører derfor udempet resonansfrekvens

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Quartz pressure sensor

$$f = \frac{1}{2\pi} \sqrt{\frac{k + \Delta k}{M}}$$



<http://www.paroscientific.com/qttechnology.htm>

Quartz microbalance



$$f = \frac{1}{2\pi} \sqrt{\frac{k}{M + \Delta m}}$$

Eksempler i boka - masse

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17 Chemical Sensors

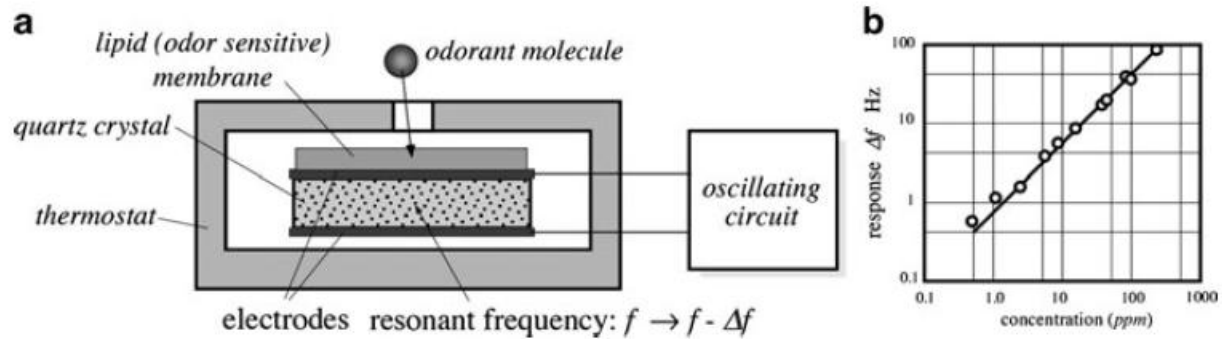


Fig. 17.14 Microbalance vapor sensor (a) and its transfer function (b) for amylacetate gas

- Kraft

Fig. 9.18 A piezoelectric disk resonator as a diametric force sensor

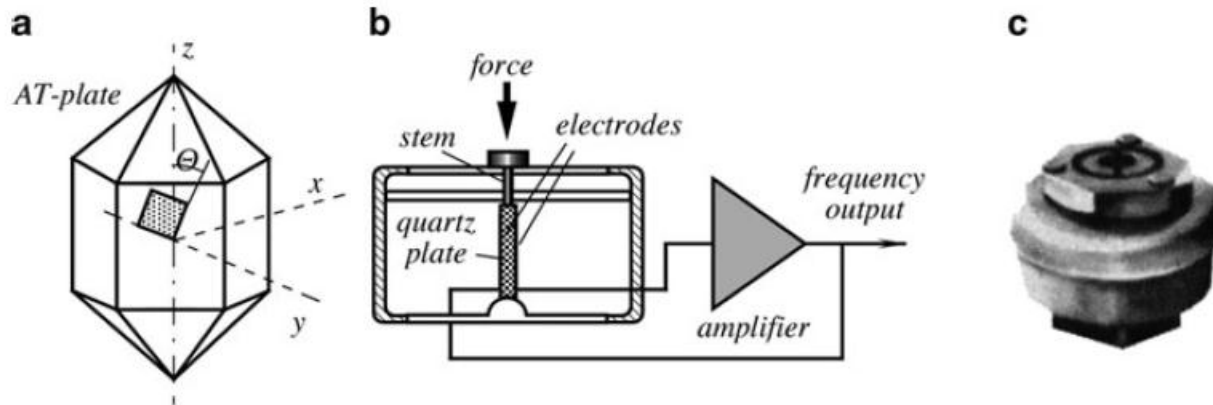
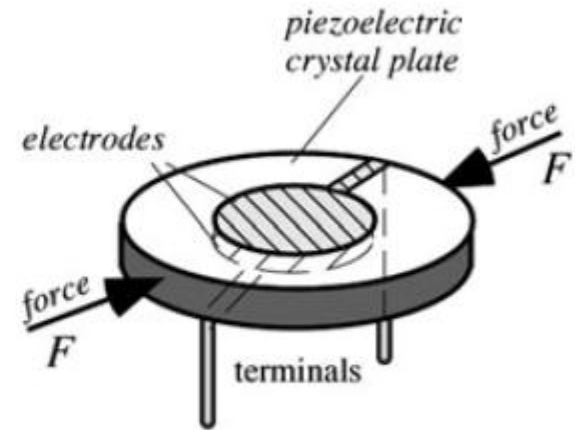


Fig. 9.19 Quartz force sensor AT-cut of a quartz crystal (a); structure of the sensor (b); outside appearance (c) (Courtesy of Quartzcell, Santa Barbara, CA)

Q-faktor

$$H_a(\omega) = \frac{\frac{1}{m}}{\omega_0^2 + \frac{j \cdot \omega_0 \cdot \omega}{Q} - \omega^2}$$

$$Q = \frac{m \cdot \omega_0}{\gamma}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$H_c(\omega) = \frac{\frac{1}{L}}{\omega_0^2 + \frac{1j \cdot \omega_0 \cdot \omega}{Q} - \omega^2}$$

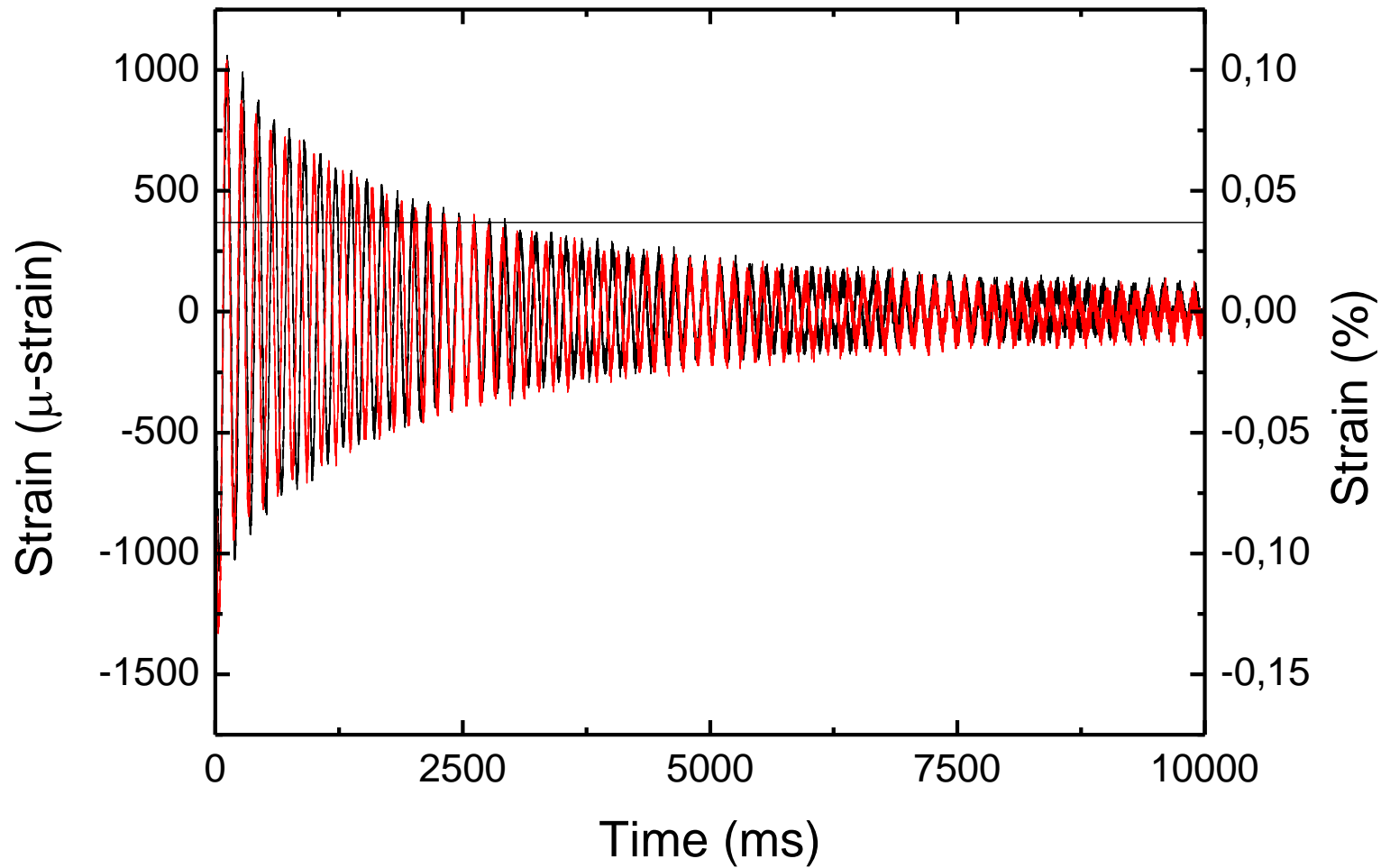
$$Q = \frac{\omega_0 \cdot L}{RC}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

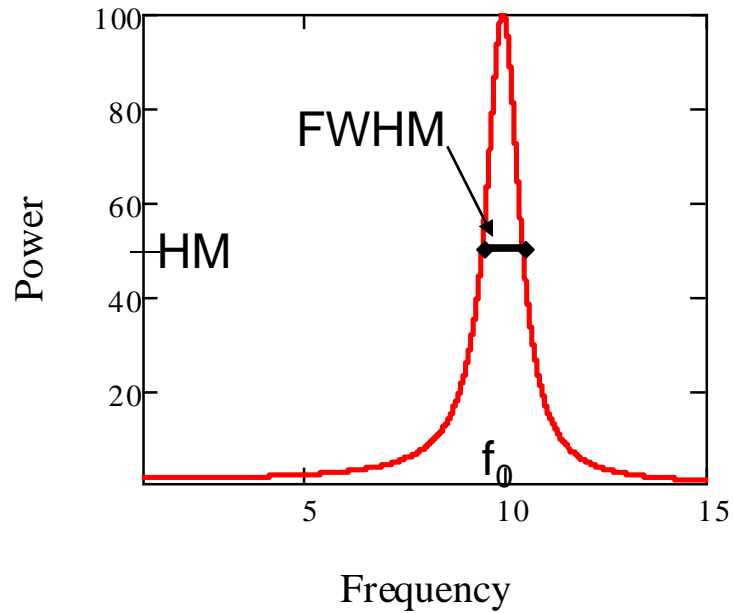
Q-factor appears as

- Stored energy divided by energy dissipated during one cycle at resonance
- Number of oscillations before the amplitude is reduced by a factor $1/e$
- Eigenfrequency divided by the Full Width at Half Maximum for the transfer function squared (power)

Q from decay



Q from power function

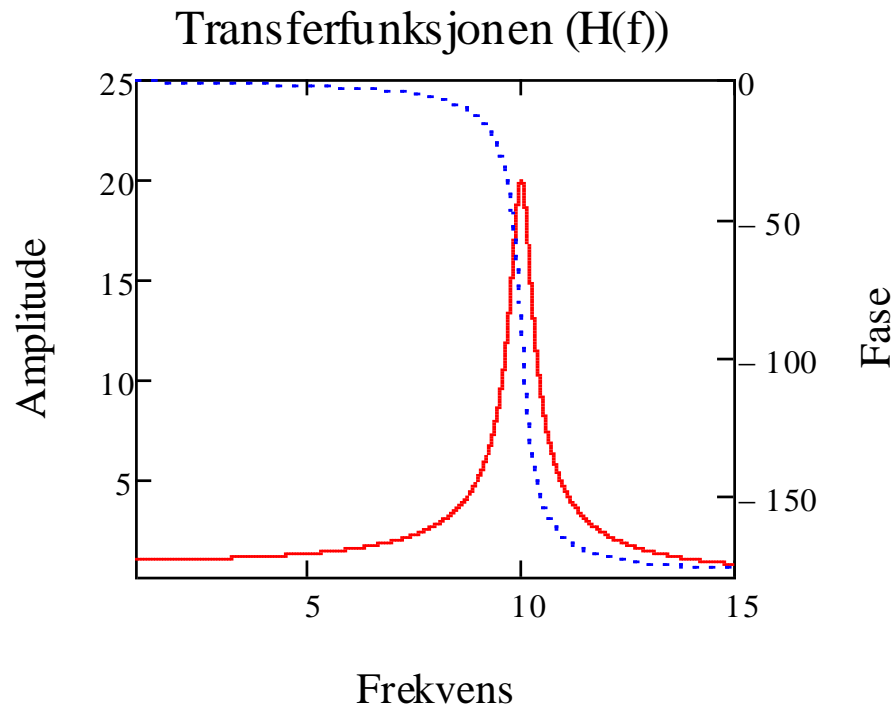


$$Q = \frac{f_0}{FWHM}$$

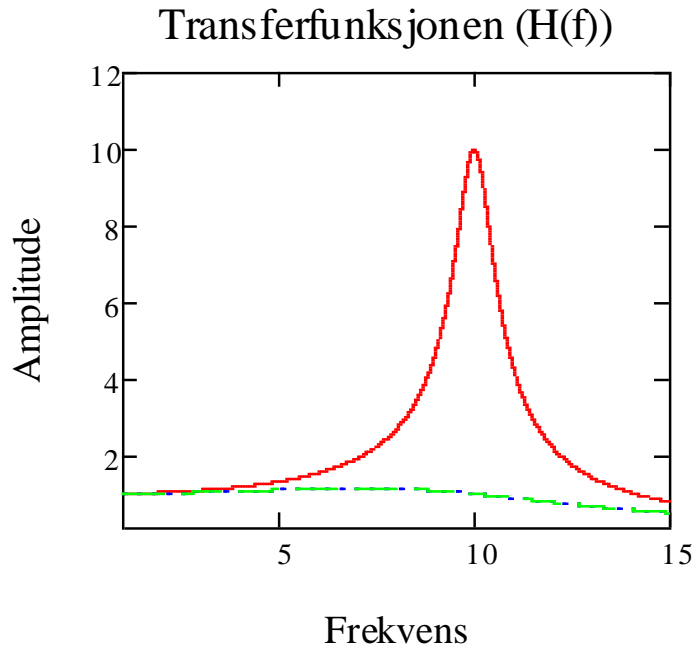
Amplitude og fase

$$|H(\omega)| = \frac{1}{m \sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \frac{\omega^2 \cdot \omega_0^2}{Q^2}}}$$

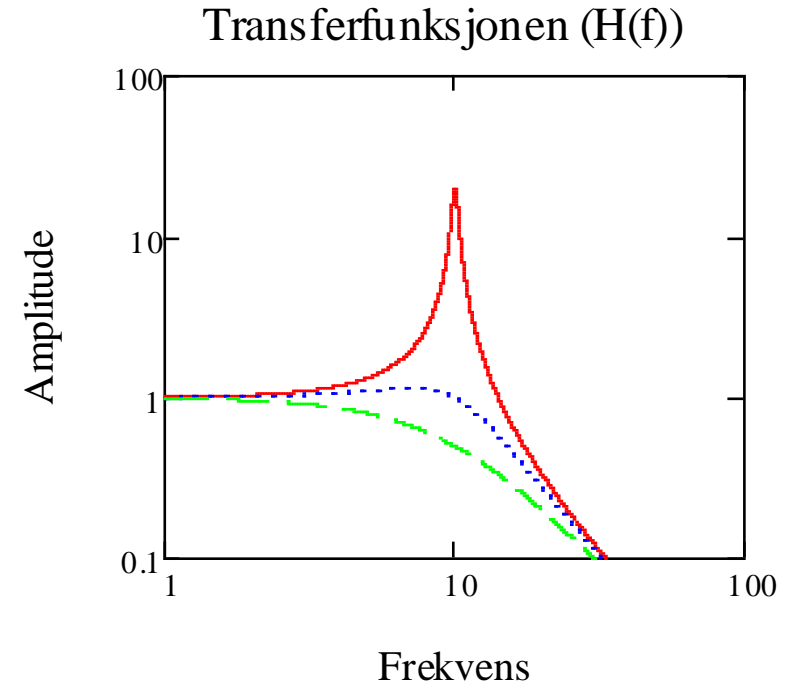
$$\theta(\omega) = \text{atan} \left[\frac{\omega \cdot \omega_0}{Q \cdot \left(\omega^2 - \omega_0^2\right)} \right]$$



Amplitude $Q=1/2, 1, 10$



Lineære akser



Logaritmiske akser

Kritisk demping (se kompendiet): $\zeta = \frac{1}{2Q} = 1$

Resonans og fase

Arbeid utført av kraften på svingesystemet

$$W = \int F dx = \int F \frac{dx}{dt} dt = \int F j\omega x dt$$

Hvis kraft og posisjon er i fase:

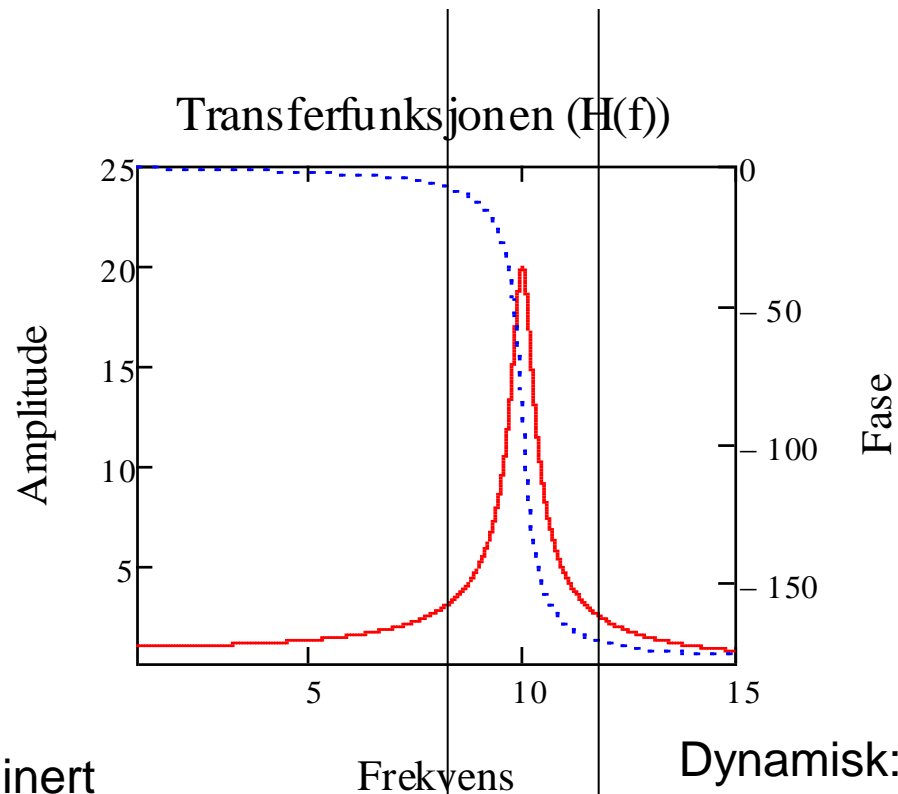
$$W = \int \operatorname{Re}(f_0 e^{j\omega t}) \operatorname{Re}(j\omega x_0 e^{j\omega t}) dt = \omega x_0 f_0 \int \cos(\omega t) \sin(\omega t) dt \approx 0$$

Hvis posisjonen henger 90 grader etter kraften: $x_0 = \frac{f_0}{j\omega_0 \gamma}$

$$W = \int \operatorname{Re}(f_0 e^{j\omega t}) \operatorname{Re}\left(j\omega \frac{f_0}{j\omega_0 \gamma} e^{j\omega t}\right) dt = \frac{1}{\gamma} f_0^2 \int \cos(\omega t) \cos(\omega t) dt \neq 0$$

Frekvens regimer

Resonans



Statisk:

Fjær dominert

Kondensator dominert

Dynamisk:

Masse dominert

Spole dominert

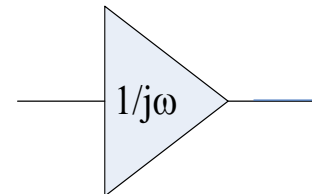
Blokkfunksjonsbeskrivelse

- Tegner alltid integrasjon

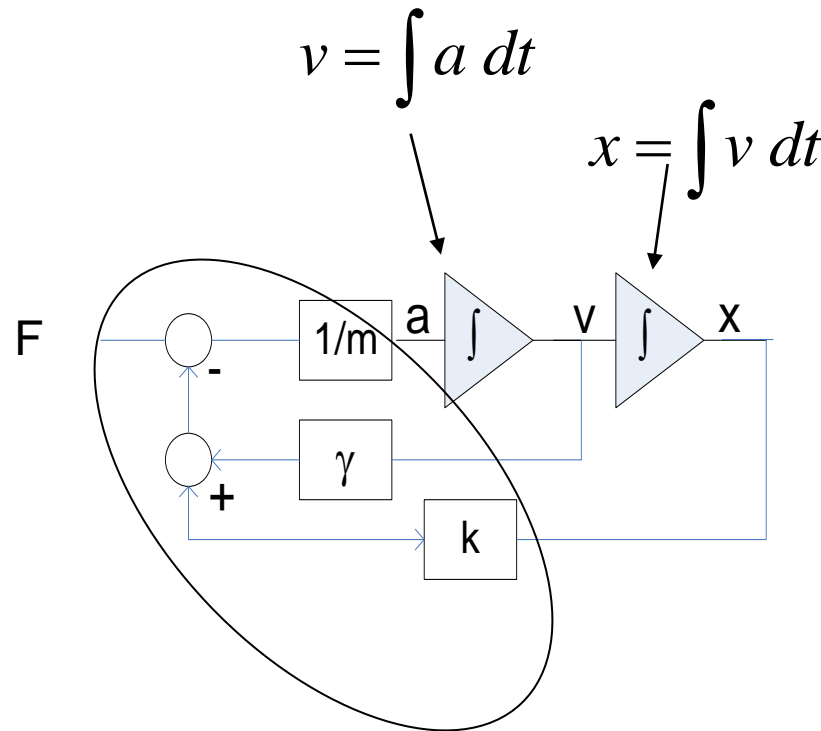


- Med antagelsen $f = C_1 e^{j\omega t}$
får vi $\int f dt = \frac{1}{j\omega} C_1 e^{j\omega t} +$

- Derfor skriver vi ofte

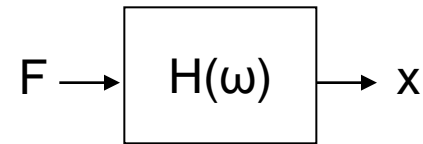
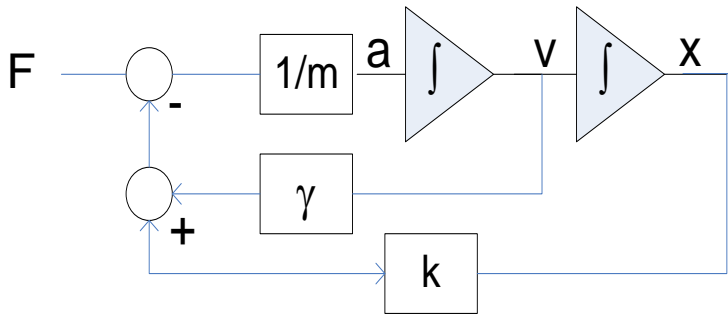


Blokkfunksjon for svingesystem



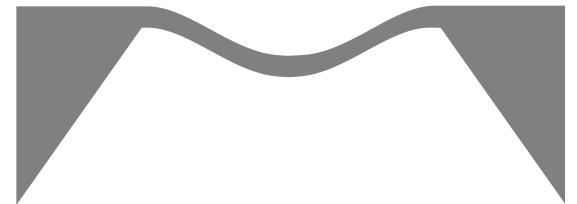
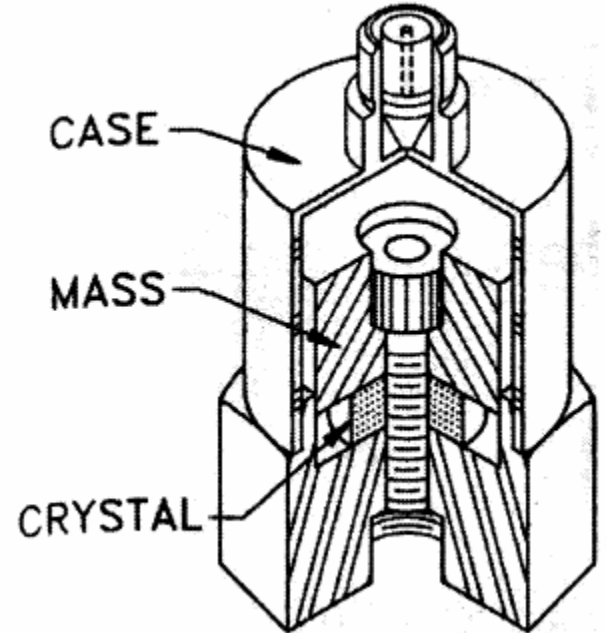
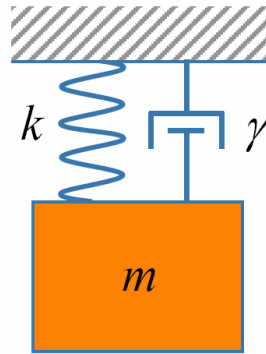
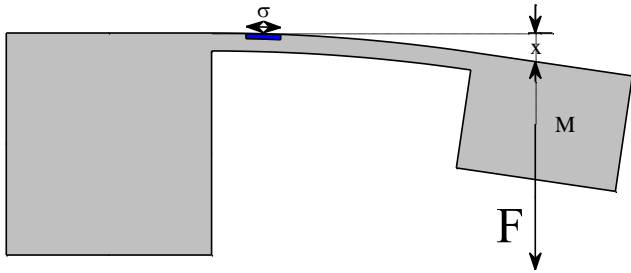
$$a = \frac{1}{m} (F - (\gamma v + kx))$$

Transferfunksjonen



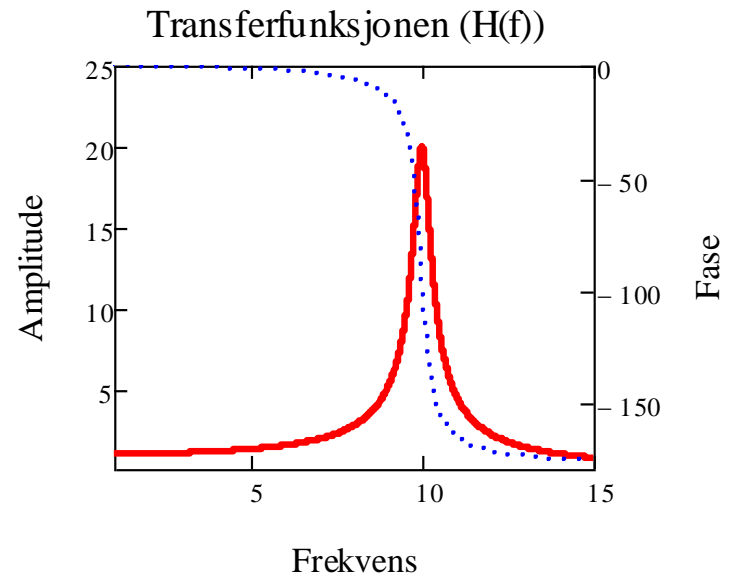
$$H(\omega) = \frac{x_0}{f_0} = \frac{1}{k + j\omega\gamma - m\omega^2}$$

Examples



Resonerende sensorer - 1

- Eksiterer et svingesystem
- Finner/følger resonanstoppene
 - Sveip
 - Faselåst sløyfe
 - Selvoscillerende krets
- Avlede størrelser som bestemmer resonansfrekvensen



Resonerende sensorer - 2

- Enkelte sensorer benytter resonans for å få økt følsomhet

Fig. 5

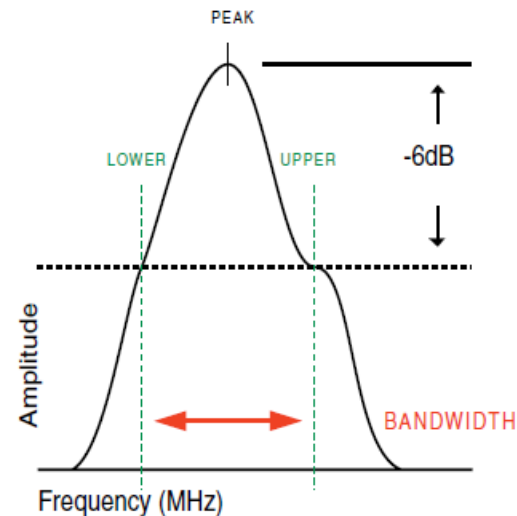
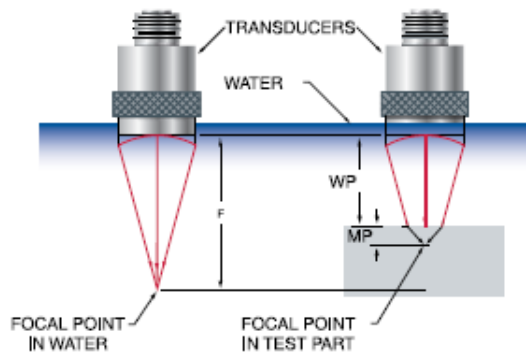









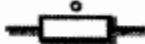




Fig. 18



Analogies

Table 3.4. Mechanical, Thermal, and Electrical Analogies

MECHANICAL	THERMAL	ELECTRICAL	
MASS  Kinetic energy $F = M \frac{d(v)}{dt}$	CAPACITANCE  C $Q = C \frac{dT}{dt}$	INDUCTOR  L Magnetic en $V = L \frac{di}{dt}$	CAPACITOR  $i = C \frac{dV}{dt}$
SPRING  k Potential energy $F = k \int v dt$	CAPACITANCE  C $T = \frac{1}{C} \int Q dt$	CAPACITOR  C Electrostatic en $V = \frac{1}{C} \int i dt$	INDUCTOR  L $i = \frac{1}{L} \int V dt$
DAMPER  b $F = bv$	RESISTANCE  R $\Delta T = RQ$ $Q = \frac{1}{R} (T_2 - T_1)$	RESISTOR  R $V = Ri$	RESISTOR  R $i = \frac{1}{R} V$

Force control

Voltage control

Current control